YIELD PREDICTION AND GROWTH PROJECTION FOR SITE-PREPARED LOBLOLLY PINE PLANTATIONS IN THE CAROLINAS, GEORGIA, ALABAMA AND FLORIDA

Plantation Management Research Cooperative

Daniel B. Warnell School of Forest Resources

University of Georgia

Athens, Georgia 30602

PMRC TECHNICAL REPORT 1996 - 1

February, 1996

Compiled by: W.M. Harrison and Bruce E. Borders

SUMMARY

This report describes an extensive set of growth and yield prediction and projection equations for site-prepared loblolly pine plantations in the Southeastern U.S. The data used to develop these models came from various studies established as much as 20 years ago. These data have determined, for the most part, the type of growth and yield systems and the specific equations which were used. In addition to what the data have determined for us, special consideration was given to the extrapolative properties and limiting relationships implied by the various models.

The growth and yield system consists of six major components. The first is a synthesis of individual-tree volume, weight and taper functions for loblolly pine. These can be used to compute inventory or research plot volumes, merchandise individual stems and create stock tables from measured or predicted stand tables.

The second component is a whole-stand growth and yield system. This system consists of equations to predict or project dominant height, trees per acre, basal area per acre and yield per acre. In addition, an equation is provided to facilitate the estimation of yields by product class.

The third component is a Weibull-based diameter distribution prediction system. This system allows for the estimation of stand tables which match the number of trees per acre and the per-acre basal area provided by inventory or by the whole-stand prediction system. In addition to the number of trees per acre by diameter class, the system includes a function to predict average heights by diameter class. The individual tree volume, weight, and/or taper equations can then be used to compute total, merchantable and product volumes.

The fourth component of the growth and yield system is a stand table projection algorithm. When a stand table is available from an inventory or from the Weibull-based system, the stand table can be projected with this method. Like the Weibull-based system, the stand table projection procedure ensures compatibility with whole-stand estimates of trees per acre and basal area per acre.

The fifth component provides growth and yield estimates for thinned plantations. This includes the estimation of thinned basal area as a function of the number of trees thinned and consideration of thinned growth response in terms of per-acre basal area. This growth response is formulated by comparing the basal area growth of a thinned plantation to the basal area growth of an unthinned counterpart of the same age, dominant height and number of trees per acre.

The final component provides adjustment functions to account for the effects of midrotation fertilization with N and P. Midrotation in this context refers to ages from 10 to 16 years. Growth response due to fertilization is accounted for in the dominant height and per-acre basal area growth equations. The predicted response is computed as a function of pounds of elemental N per acre, whether or not P was applied, and the number of years since treatment.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

The most recent growth and yield models for site-prepared loblolly pine plantations developed at the Daniel B. Warnell School of Forest Resources were published in 1990 by Borders, et.al. and in 1994 by Borders. These models have been implemented and critiqued by members of the Plantation Management Research Cooperative (PMRC). The need to revise the loblolly growth and yield model was expressed through the comments of various PMRC cooperators. This report describes a revised loblolly model based on all available data to date.

The model structure is basically unchanged from the previous versions. For most model components, separate coefficient estimates were obtained for the combined Piedmont and Upper Coastal Plain data and for the Lower Coastal Plain data. In all three physiographic regions, some plots were found to have experienced excessive mortality over 4-year measurement intervals. An investigation of field data forms provided sufficient justification to exclude some of these plots. Plots were excluded due to fire damage, beetle damage, wind damage, harvest damage, thinning, change in plot layout and/or an excessive number of small wildlings. A total of 23 plots in the Lower Coastal Plain, 20 plots in the Piedmont and 5 plots in the Upper Coastal Plain were excluded from the modelling exercise.

As in previous versions, the loblolly model consists of three alternative yield simulation systems:

- A diameter distribution system based on the Weibull distribution,
- A stand table projection system,
- A whole-stand yield prediction and projection system.

Each of these three systems is "driven" by whole-stand prediction and projection models for average dominant height, trees per acre and basal area per acre. In order to complete a full set of loblolly growth and yield models, a description of the individual tree volume, weight and taper equations developed by Pienaar et.al. (1987) is included.

The basic set of loblolly growth and yield models was developed based on plots which have not been thinned, fertilized or weeded. The impact of hardwood competition has been included in this model revision for the Piedmont region. Remeasurement data on hardwoods have been obtained from the last two measurements of the Piedmont growth and yield plots. The MS33 thinning study data were used to assess the growth of thinned plantations. Data from the NC State Regionwide 13 fertilizer study were used to take a preliminary look at growth response due to midrotation fertilization.

2 NOTATION

The following notation will be used throughout this document:

A_i = age (years) at time i,

D_{qi} = quadratic mean diameter (inches) at time i,

HD_i	=	average dominant height (ft) at time i,
TPA_i	=	trees per acre at time i,
SI_{25}	=	base age 25 site index (ft),
BA_i	=	per acre basal area (ft^2) at time i,
$TVOB_i$	=	total volume per acre outside bark (ft^3) at time i,
$TVIB_i$	=	total volume per acre inside bark (ft^3) at time i,
$GWOB_i$	=	total green weight per acre outside bark (tons) at time i,
$DWIB_i$	=	total dry weight per acre inside bark (tons) at time i,
Dbh	=	tree diameter at breast height (inches),
H	=	total tree height (ft),
D_m	=	merchantable top diameter (inches),
M	=	height (ft) to diameter limit D_m,
VOB_m	=	stem volume outside bark to a top diameter of D_m inches (ft^3),
VIB_m	=	stem volume inside bark to a top diameter of D_m inches (ft^3),
$GWWB_m$	=	stem green weight with bark to a top diameter of D_m inches (lbs),
$GWIB_m$	=	stem green weight without bark to a top diameter of D_m inches (lbs),
DW_m	=	stem dry weight without bark to a top diameter of D_m inches (lbs).

3 PMRC GROWTH AND YIELD DATA

The data used in this modelling exercise come from the PMRC loblolly growth and yield plots established and remeasured over the past 19 years. The first series of plots was established in the coastal plain of North and South Carolina in 1977 and was remeasured in 1981. The second database is composed of plots installed in the lower coastal plain of Georgia and north Florida in 1981 and remeasured in 1985 and 1989. These two datasets were combined to form the Lower Coastal Plain database which has a total of 606 plots.

A total of 199 permanent plots was established in the Piedmont region of Alabama, Georgia and South Carolina beginning in 1982. These plots were remeasured in 1987 and in 1991. One hundred sixteen permanent plots were installed in the upper coastal plain region of Alabama, Georgia and South Carolina in 1981. These plots were remeasured in 1986 and in 1991. The three physiographic regions used to stratify the plot sample are shown in Figure 1.

A detailed description of plot layout and data collection procedures for these plots is provided by Bailey, et.al. (1985) and by Borders, et.al. (1990). Tables 1-3 show the distribution of sample measurements by site index, density and age

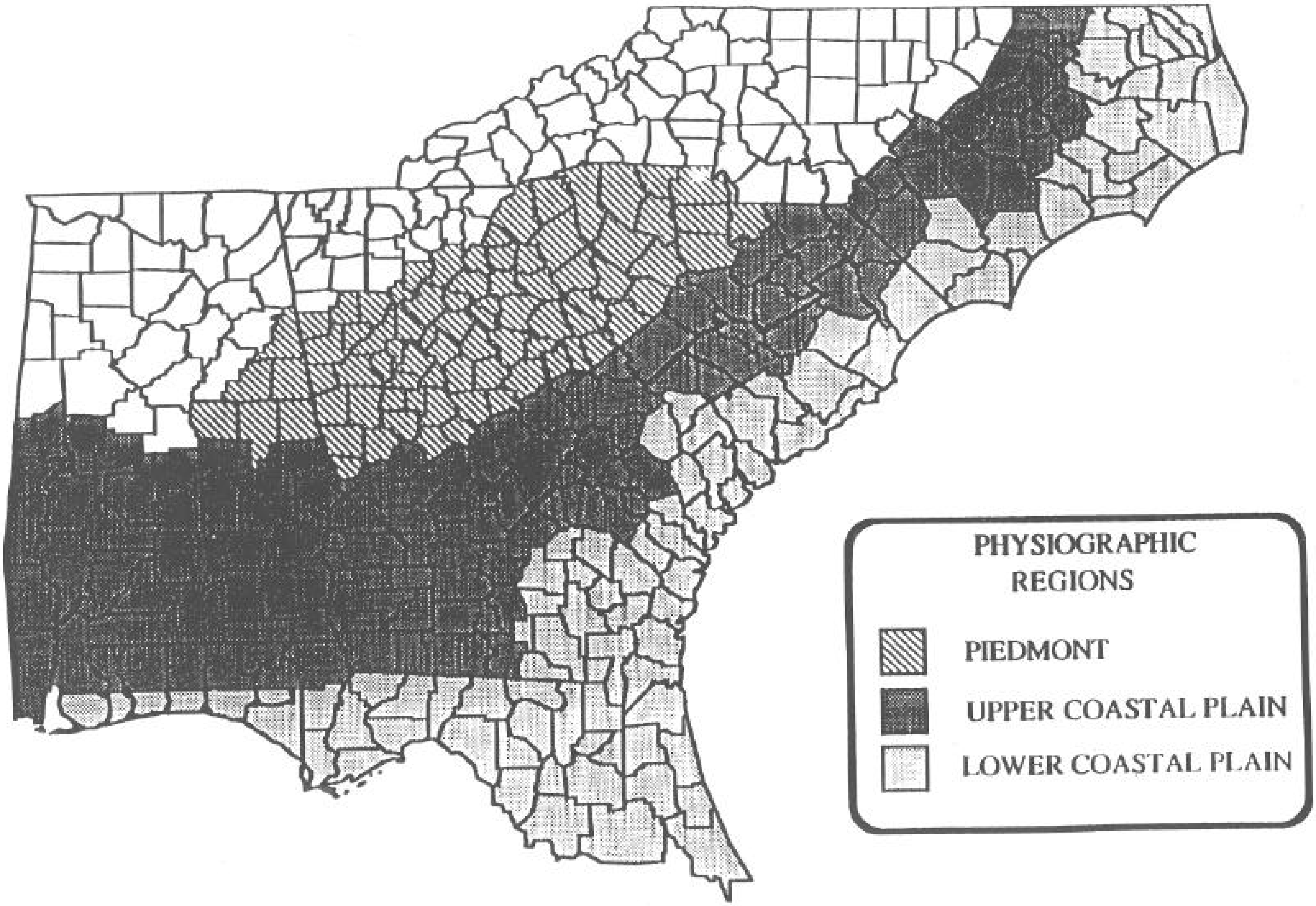

Figure 1. Three physiographic regions defined across the Carolinas, Georgia, Florida and Alabama.

classes. The samples are reasonably well distributed although data in the older age classes and higher density classes are lacking.

Table 1: Number of sample measurements by region and site index class.

Site Index Class (ft)	Lower Coastal Plain	Upper Coastal Plain	Piedmont
30	4	5	0
40	41	31	40
50	119	76	232
60	208	105	170
70	164	59	37
80	67	10	5
90	3	3	0
TOTAL	606	289	484

Table 2: Number of sample measurements by region and density class.

Density Class (TPA)	Lower Coastal Plain	Upper Coastal Plain	Piedmont
100	2	0	1
200	7	6	18
300	47	18	58
400	143	55	112
500	170	99	115
600	117	52	81
700	63	29	46
800	34	17	35
900	19	11	8
1000	3	2	6
1100	1	0	3
1200	0	0	3
1300	0	0	0
1400	0	0	1
TOTAL	606	289	487

Table 3: Number of sample measurements by region and age class.

Age Class (years)	Lower Coastal Plain	Upper Coastal Plain	Piedmont
10	86	41	51
15	240	100	161
20	198	111	161
25	68	33	85
30	13	3	24
35	1	1	5
TOTAL	606	289	487

Tables 4-5 show summary statistics for Lower Coastal Plain measurements at initial and subsequent measurement periods. Tables 6-7 show statistics for the Piedmont data and tables 8-9 show statistics for the Upper Coastal Plain data.

Table 4: Summary statistics for initial measurements of Lower Coastal Plain plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_1	254	15.78	9.0	30.67	3.97
TPA_1	254	562.2	150	1140	162.36
HD_1	254	46.37	19.2	77	11.74
BA_1	254	122.7	14.1	244	38.69
$TVOL_1$	254	2524	36.1	6584	1464
$TVIB_1$	254	2028	26.2	5580	1230
$GWOB_1$	254	71.7	1.00	189.1	42.0
$DWIB_1$	254	28.8	0.33	87.6	18.5
% $Cronartium_1$	254	21.5	0	73.8	15.3

Table 5: Summary statistics for remeasurements of Lower Coastal Plain plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_2	254	19.74	12.5	34.83	4.01
TPA_2	254	501.86	130	916	149.6
HD_2	254	54.68	26.5	87.36	12.15
BA_2	254	139.29	38.9	234.3	33.40
$TVOL_2$	254	3464	364	7399	1534
$TVIB_2$	254	2832	270	6356	1321
$GWOB_2$	254	98.7	10.2	213.1	44.2
$DWIB_2$	254	41.7	3.7	111.5	20.5
% $Cronartium_2$	254	22.0	0	77.8	13.5

Table 6: Summary statistics for initial measurements of Piedmont plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_1	281	16.84	9.67	29.83	4.44
TPA_1	281	541.4	164.7	1372.1	190.6
HD_1	281	41.4	23.5	75.22	10.47
BA_1	281	96.1	24.7	210.1	32.92
$TVOL_1$	281	1795	34.3	6008	1125
$TVIB_1$	281	1430	24.9	5065	949
$GWOB_1$	281	46.6	0.80	166.3	31.0
$DWIB_1$	281	18.8	0.31	69.4	13.1
% $Cronartium_1$	281	34.2	0.0	76.3	16.65

Table 7: Summary statistics for remeasurements of Piedmont plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_2	281	21.09	13.5	34.67	4.55
TPA_2	281	494.0	147.4	1230	171.7
HD_2	280	49.28	27.15	83.83	10.27
BA_2	281	116.20	36.5	206	29.62
$TVOL_2$	281	2569	356.3	6576	1117
$TVIB_2$	281	2086	262.00	5661	970
$GWOB_2$	281	68.1	8.46	186.0	31.8
$DWIB_2$	281	28.2	3.41	75.6	13.7
% $Cronartium_2$	281	35.0	0.0	78.1	17.0

Table 8: Summary statistics for initial measurements of Upper Coastal Plain plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_1	173	15.48	10.67	28.67	3.30
TPA_1	173	570.3	233.9	985.5	160.8
HD_1	173	42.01	19.5	72.05	10.78
BA_1	173	102.8	16.9	207.3	32.69
$TVOL_1$	173	2030	0	6136	1223
$TVIB_1$	173	1618	0	5082	1018
$GWOB_1$	173	53.6	0	166.8	33.6
$DWIB_1$	173	21.2	0	70.5	14.2
% $Cronartium_1$	173	41.1	1.6	76.1	19.4

Table 9: Summary statistics for remeasurements of Upper Coastal Plain plots.

Variable	# observations	Mean	Minimum	Maximum	Standard Deviation
A_2	173	19.81	14.5	33.83	3.33
TPA_2	173	520.1	185.1	903.4	150.8
HD_2	173	50.26	23.09	85.33	11.56
BA_2	173	122.79	25.10	212.4	32. 28
$TVOL_2$	173	2905	27.9	6759	1343
$TVIB_2$	173	2353	20.4	5655	1141
$GWOB_2$	173	77.7	0.7	185.9	37.6
$DWIB_2$	173	32.1	0.2	80.9	16.5
% $Cronartium_2$	173	38.6	1.7	89.2	19.7

4 INDIVIDUAL TREE VOLUME, WEIGHT AND TAPER FUNCTIONS

In order to compute plot volumes and volume estimates from stand tables, individual tree volume, weight and taper functions are required. Pienaar, et.al. (1987) developed individual tree equations for loblolly pine in the three physiographic regions described previously. These equations are listed below for convenient reference.

4.1 Outside bark stem volume and taper

The equation forms for outside bark volume and taper are shown below. Parameter estimates for the three physiographic regions are listed in Table 10.

$$VOB_m = b_0 \, Dbh^{b_1} H^{b_2} - b_3 \left[\frac{D_m^{b_4}}{Dbh^{b_4 - 2}} \right] (H - 4.5) \qquad \textbf{(1)}$$

$$D_m = Dbh \left[\frac{H - M}{H - 4.5} \right]^{\frac{1}{b_4 - 2}} \qquad \textbf{(2)}$$

$$M = H - (H - 4.5)\left[\frac{D_m}{Dbh}\right]^{b_4 - 2} \qquad \textbf{(3)}$$

Table 10: Parameter estimates by physiographic region for outside bark stem volume and taper.

Region	b_0	b_1	b_2	b_3	b_4
Lower Coastal Plain	0.00145519	1.826051	1.221965	0.00253872	3.741575
Upper Coastal Plain	0.00431899	1.953207	0.896934	0.00251744	3.714466
Piedmont	0.00401246	1.829011	0.969142	0.00249374	3.684725

4.2 Inside bark stem volume and taper

The equation forms for inside bark volume and taper are shown below. Parameter estimates for the three physiographic regions are listed in Table 11.

$$VIB_m = b_0 \; Dbh^{b_1} H^{b_2} - b_3 \left[\frac{D_m^{b_4}}{Dbh^{b_4 - 2}}\right](H - 4.5) \qquad \textbf{(4)}$$

$$D'_m = \left[b_5 \; DBH^2 \left(\frac{H - M}{H - 4.5}\right)^{b_6}\right]^{1/2} \qquad \textbf{(5)}$$

where $\mathbf{D'_m}$ = inside bark diameter in inches where the outside bark diameter is D_m inches.

Table 11: Parameter estimates by physiographic region for inside bark stem volume and taper.

Region	b_0	b_1	b_2	b_3	b_4	b_5	b_6
Lower Coastal Plain	0.00071193	1.876991	1.321458	0.00217131	3.592491	0.821198	1.062783
Upper Coastal Plain	0.00210741	1.957418	1.021763	0.00209273	3.584111	0.802118	1.090512
Piedmont	0.00171199	1.870407	1.110322	0.00210729	3.437603	0.788358	1.040453

4.3 Green weight with bark

The equation form for green weight with bark is shown below. Parameter estimates for the three physiographic regions are listed in Table 12.

$$GWWB_m = b_0\, Dbh^{b_1} H^{b_2} - b_3 \left[\frac{D_m^{b_4}}{Dbh^{b_5}} \right] (H - 4.5) \qquad \textbf{(6)}$$

Table 12: Parameter estimates by physiographic region for green weight with bark.

Region	b_0	b_1	b_2	b_3	b_4	b_5
Lower Coastal Plain	0.0740959	1.829983	1.247669	0.123329	3.523107	1.449947
Upper Coastal Plain	0.141534	1.917146	1.038452	0.0932063	3.589155	1.413061
Piedmont	0.110069	1.935455	1.080621	0.0775771	3.439954	1.178473

4.4 Dry weight without bark

The equation form for dry weight without bark is shown below. Parameter estimates for the three physiographic regions are listed in Table 13.

$$DW_m = b_0\, Dbh^{b_1} H^{b_2} - b_3 \left[\frac{D_m^{b_4}}{Dbh^{b_5}} \right] (H - 4.5) \qquad \textbf{(7)}$$

Table 13: Parameter estimates by physiographic region for dry weight without bark.

Region	b_0	b_1	b_2	b_3	b_4	b_5
Lower Coastal Plain	0.0106276	1.882913	1.478766	0.0298084	3.825425	1.517983
Upper Coastal Plain	0.0290299	2.017530	1.157743	0.0222220	3.782287	1.367710
Piedmont	0.0360196	1.742939	1.232462	0.0356069	3.668307	1.479158

If tree age is known, an alternative form of the dry weight without bark equation is available. The equation form is shown below and parameter estimates by region are shown in Table 14.

$$DW_m = b_0 \, Dbh^{b_1} H^{b_2} A^{b_3} - b_4 \left[\frac{D_m^{b_5}}{Dbh^{b_6}} \right] (H - 4.5) \qquad \textbf{(8)}$$

Table 14: Parameter estimates by physiographic region for dry weight without bark when tree age is known.

Region	b_0	b_1	b_2	b_3	b_4	b_5	b_6
Lower Coastal Plain	0.0113113	1.901901	1.303882	0.210461	0.0309330	3.821368	1.526992
Upper Coastal Plain	0.0275683	1.973518	1.093663	0.137418	0.0217837	3.769104	1.345945
Piedmont	0.0288583	1.769315	1.161088	0.154501	0.0363042	3.654891	1.474768

4.5 Green weight without bark

In the Piedmont region, green weight without bark can be estimated using the following equation:

$$GWIB_m = 0.120931 \, Dbh^{2.323008} H^{0.823979} - 0.076815 \left[\frac{D_m^{3.446656}}{Dbh^{1.238789}} \right] (H - 4.5) \qquad \textbf{(9)}$$

5 WHOLE STAND MODELS

5.1 Dominant height and site index functions

The most consistent and useful measure of site quality for modelling purposes is site index. In this context, site index is defined as the average height of dominant and codominant trees at base age 25 years. For site-prepared loblolly pine plantations, Pienaar and Shiver (1980) developed site curves for soil groups referred to as A and B. Soil group B consists of soils in North Carolina pocosin river swamps that have been ditched and drained. The soil series include Ballah, Torhunta, Bayboro, Pantego and Byars. No additional data have been obtained in these areas since the site curves were initially developed, thus we continue to rely on Pienaar and Shiver's equations which are:

$$SI_{25} = HD\left[\frac{0.7476}{1-e^{-0.05507\ A}}\right]^{1.435} \tag{10}$$

$$HD = SI_{25}\left[\frac{0.7476}{1-e^{-0.05507\ A}}\right]^{-1.435} \tag{11}$$

A dominant height projection equation was developed from the PMRC loblolly data representing soil group A (all data not classified as representing soil group B). Several equation forms were evaluated, including the form developed by Clutter and Jones (1980) and subsequently used in previous PMRC loblolly growth and yield systems (Borders, et.al., 1990; Borders, 1994). The Chapman-Richards height growth model, however, was found to result in a superior fit for the PMRC loblolly data. A conditional F-test revealed that the same height projection model could be used in all three physiographic regions. The projection model, site index equation and height prediction model are shown below:

$$HD_2 = HD_1\left[\frac{1-e^{-0.014452\ A_2}}{1-e^{-0.014452\ A_1}}\right]^{0.8216} \tag{12}$$

n = 628 R^2 = 0.94 $S_{y.x}$ = 2.80 ft.

$$SI_{25} = HD\left[\frac{0.30323}{1-e^{-0.014452\ A}}\right]^{0.8216} \tag{13}$$

$$HD = SI_{25}\left[\frac{0.30323}{1-e^{-0.014452\ A}}\right]^{-0.8216} \tag{14}$$

Site index curves resulting from equation **(14)** are shown in Figure 2.

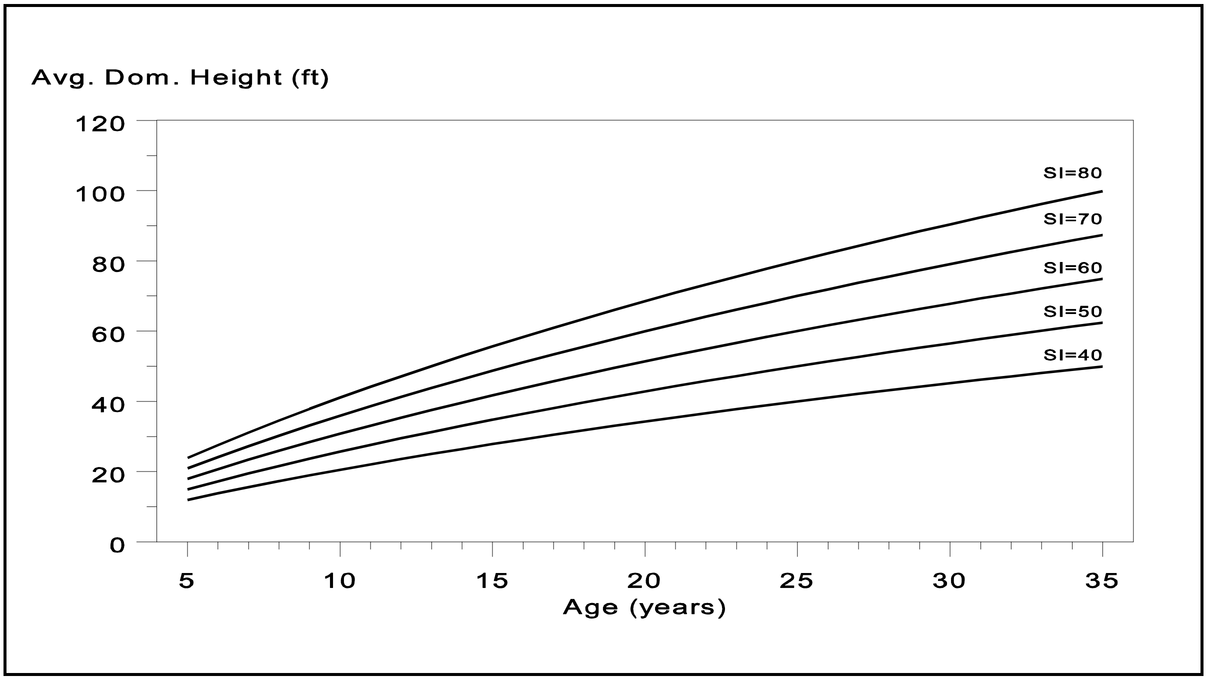

Figure 2. Loblolly pine site index curves for all physiographic regions.

5.2 Survival function

Several equation forms were evaluated as to suitability for survival prediction for the PMRC loblolly dataset. As in the Borders (1994) report, the modified Clutter and Jones (1980) equation resulted in a superior fit. This model, however, produced unrealisitc results in simulation tests. When projected past the range of the PMRC data, the projected rate of mortality remained essentially constant for a given site index and initial number of trees per acre. In order to overcome this problem, a survival equation including a specified asymptotic number of trees per acre was developed. A range of asymptotes was evaluated with the objective of achieving reasonable goodness-of-fit within the range of data while maintaining desirable extrapolative properties. This was achieved with an asymptotic survival of 100 trees per acre. A conditional F-test revealed no significant differences in survival equation parameter estimates for the three physiographic regions. The resulting survival prediction equation is:

$$TPA_2 = 100 + \left[(TPA_1 - 100)^{-0.745339} + 0.0003425^2\, SI_{25} \, (A_2^{1.97472} - A_1^{1.97472}) \right]^{-\frac{1}{0.745339}} \qquad \textbf{(15)}$$

$n = 569 \qquad R^2 = 0.95 \qquad S_{y.x} = 31.8$ TPA

With the lower asymptotic survival of 100 trees per acre, caution must be exercised in the implementation of the survival function. If the initial density (TPA_1) of a stand is 100 trees per acre or less, equation **(15)** cannot be used. It may be reasonable to assume that stands with an initial density of 100 trees per acre or less would either not experience additional mortality, or would assume a specified constant survival rate.

The implied survival trends for age five densities of 300, 500 and 700 trees per acre and a site index of 60 feet are shown in Figure 3. Figure 4 shows survival trends for different site indices given the same initial density. As the model form implies, the rate of mortality increases with increasing site index.

5.3 Basal area prediction and projection

To obtain accurate prediction and/or projection of per acre yield, it is necessary to use both the number of trees per acre and the per acre basal area as measures of stand density. When an estimate of current basal area per acre is needed and current age, trees per acre and dominant height are known, a basal area prediction model of the form of equation **(16)** is required. When current basal area is known along with current and future age, trees per acre and dominant height, a model of the form of equation **(17)** can be used to project the future basal area per acre.

$$\ln(BA) = b_0 + \frac{b_1}{A} + b_2 \ln(TPA) + b_3 \ln(HD) + b_4 \frac{\ln(TPA)}{A} + b_5 \frac{\ln(HD)}{A} \qquad \textbf{(16)}$$

$$\begin{aligned}\ln(BA_2) = {} & \ln(BA_1) + b_1\left[\frac{1}{A_2} - \frac{1}{A_1}\right] + b_2\,[\ln(TPA_2) - \ln(TPA_1)] + b_3\,[\ln(HD_2) - \ln(HD_1)] \\ & + b_4\left[\frac{\ln(TPA_2)}{A_2} - \frac{\ln(TPA_1)}{A_1}\right] + b_5\left[\frac{\ln(HD_2)}{A_2} - \frac{\ln(HD_1)}{A_1}\right] \qquad \textbf{(17)}\end{aligned}$$

These equations were fit to the PMRC loblolly database. A simultaneous fitting procedure was used to ensure compatibility between the basal area prediction and projection equations. A conditional F-test on error sum of squares from the basal area prediction equation revealed significant differences among physiographic regions. Therefore, separate parameter estimates were obtained for the Lower Coastal Plain region and for the combined Piedmont and Upper Coastal Plain regions. The parameter estimates and fit statistics by region are shown in Tables 15 and 16, respectively.

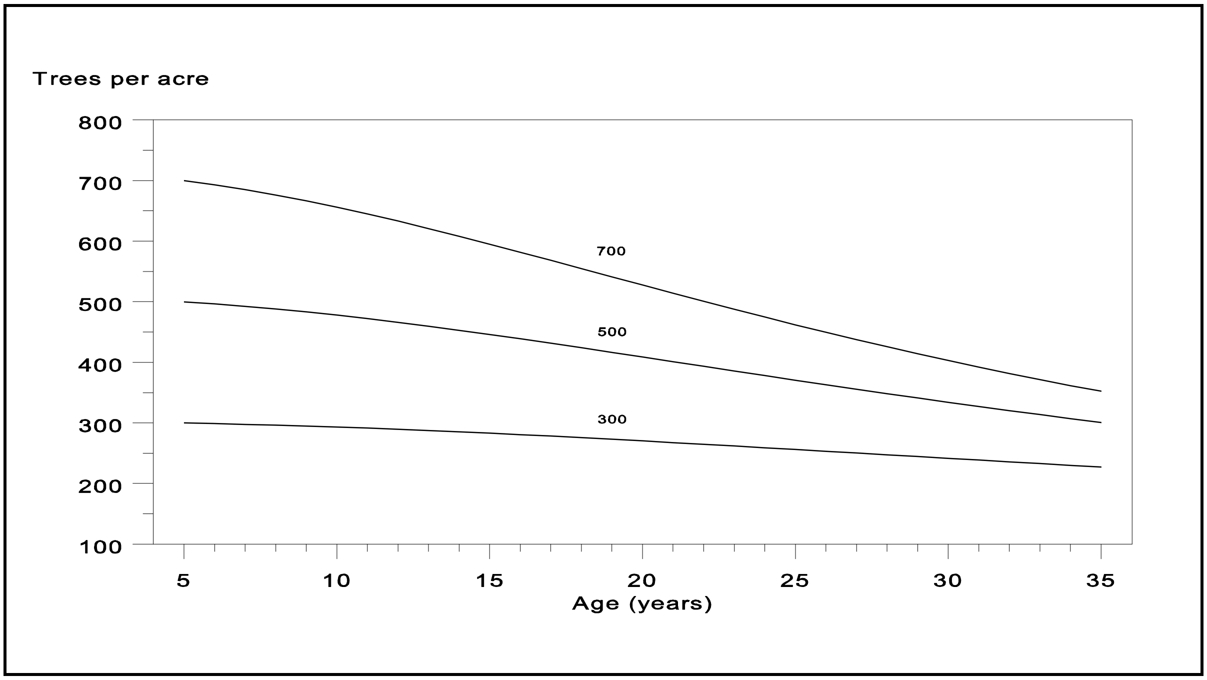

Figure 3. Survival curves for a site index of 60 feet and age five densities of 300, 500 and 700 trees per acre in all physiographic regions.

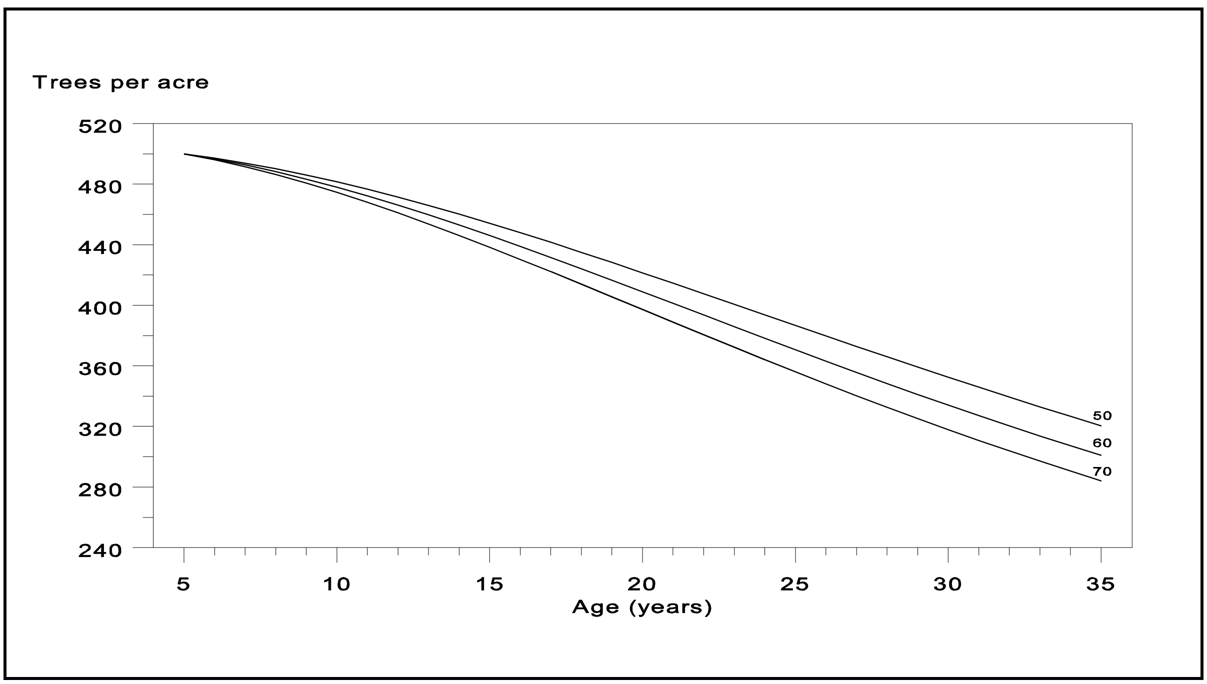

Figure 4. Survival curves for an age five density of 500 trees per acre and site indices of 50, 60 and 70 feet in all physiographic regions.

Table 15: Parameter estimates by physiographic region for per acre basal area prediction and projection.

Region	b_0	b_1	b_2	b_3	b_4	b_5
Lower Coastal Plain	0.0	-42.689283	0.367244	0.659985	2.012724	7.703502
Upper Coastal Plain + Piedmont	-0.855557	-36.050347	0.299071	0.980246	3.309212	3.787258

Table 16: Fit statistics by physiographic region for per acre basal area prediction and projection.

Region	n	Prediction		Projection	
		R^2	$S_{y.x}$	R^2	$S_{y.x}$
Lower Coastal Plain	396	0.78	0.1396	0.74	0.1275
Upper Coastal Plain + Piedmont	735	0.83	0.1440	0.78	0.1378

Figures 5 and 6 show basal area development curves for different densities and site indices in the Piedmont and Upper Coastal Plain regions. Figures 7 and 8 show basal area curves for the Lower Coastal Plain region.

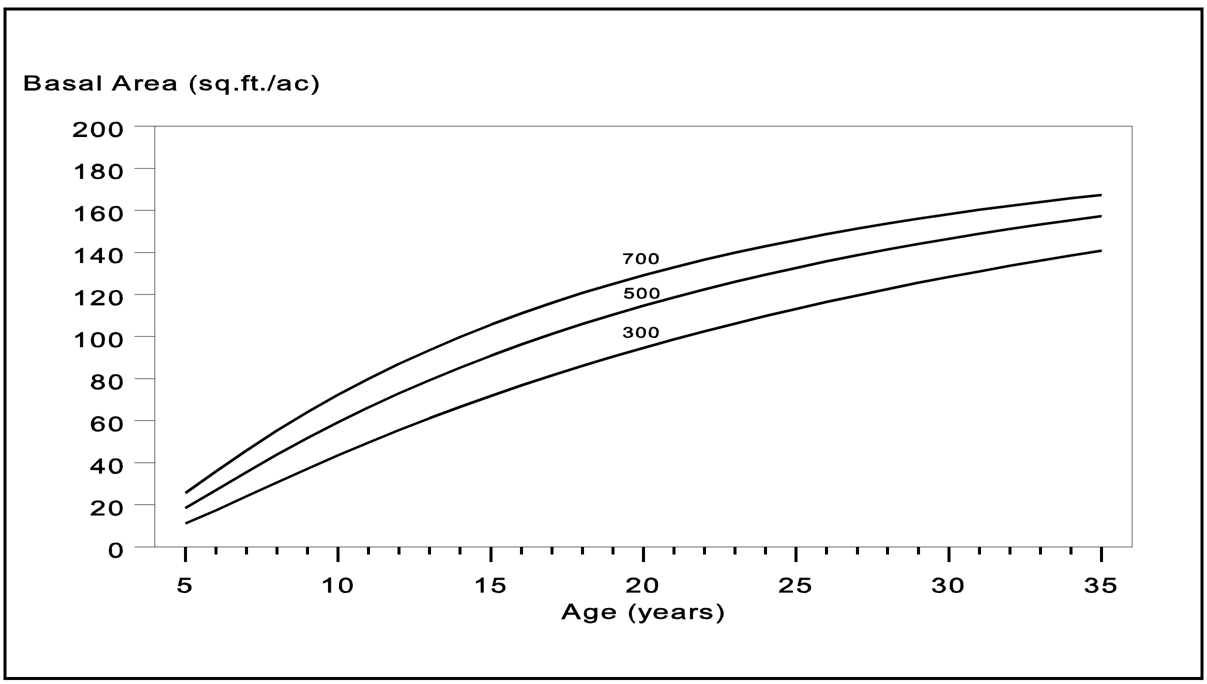

Figure 5. Basal area growth curves for a site index of 60 feet and age five densities of 300, 500 and 700 trees per acre in the Piedmont and Upper Coastal Plain regions.

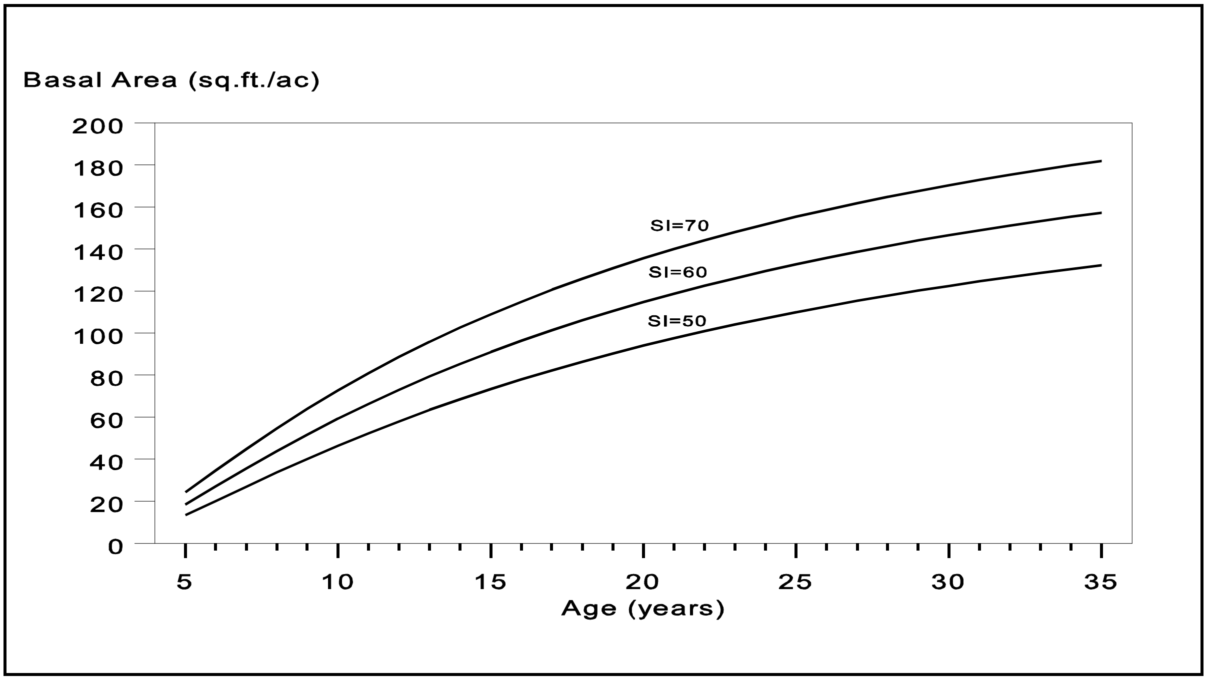

Figure 6. Basal area growth curves for an age five density of 500 trees per acre and site indices of 50, 60 and 70 feet in the Piedmont and Upper Coastal Plain regions.

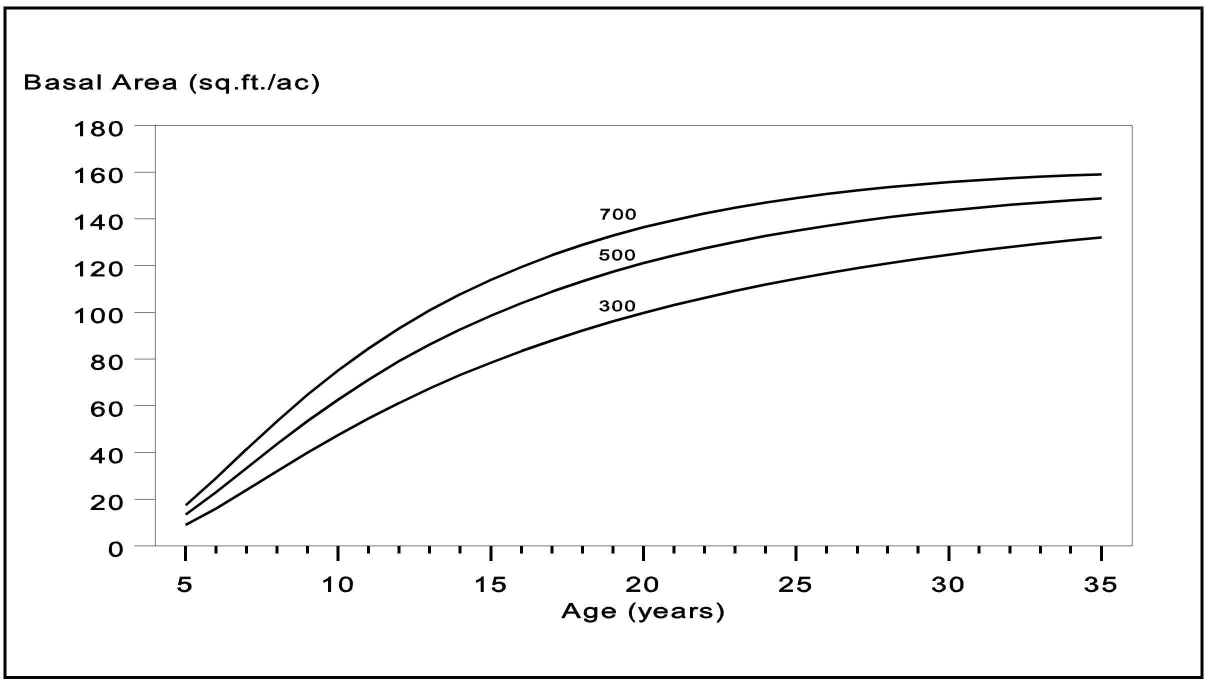

Figure 7. Basal area growth curves for a site index of 60 feet and age five densities of 300, 500 and 700 trees per acre in the Lower Coastal Plain region.

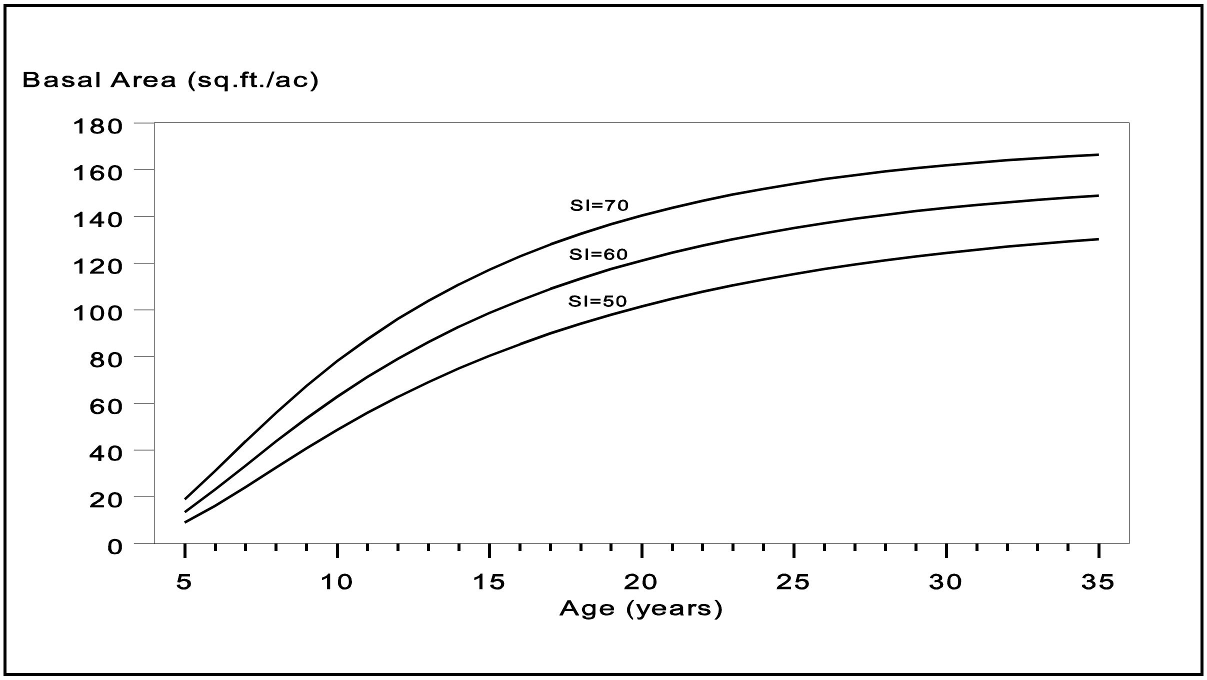

Figure 8. Basal area growth curves for an age five density of 500 trees per acre and site indices of 50, 60 and 70 feet in the Lower Coastal Plain region.

In the Piedmont region, hardwood trees growing on the PMRC loblolly growth and yield plots were measured in the last two data collection exercises. These measurements were used to assess the effect of the hardwood component on pine basal area growth. Hardwood competition was accounted for in terms of the basal area of hardwood stems greater than 2" Dbh expressed as a percentage of pine basal area (PHWD). This quantity proved to be a significant independent variable in the following loblolly basal area prediction equation for the Piedmont region:

$$\ln(BA) = -0.904066 - \frac{33.811815}{A} + 0.321301\,\ln(TPA) + 0.985342\,\ln(HD) + 3.381071\,\frac{\ln(TPA)}{A} + 2.548207\,\frac{\ln(HD)}{A} - 0.003689\,PHWD \qquad \textbf{(18)}$$

n = 559 $\quad R^2 = 0.82 \quad S_{y.x} = 0.1441$

Figure 9 shows basal area growth curves produced with equation **(18)** for various levels of hardwood competition and, for comparison, a basal area growth curve produced using equation **(16)**. Figure 10 illustrates the implications of changes over time in the level of hardwood competition. The graph shows basal area development curves for a stand which had no hardwood component, a stand which had 25% hardwood from age 10 to 30, a stand where the hardwood component increased from 10% to 20% and a stand where the hardwood component decreased from 20%

to 10% between the ages of 10 and 30. In the PMRC Piedmont loblolly data, the percentage of hardwood tended to slightly increase over time. An unsuccessful attempt was made to model the change in hardwood competition over four-year growth intervals. More data of this type will be required to model the change in hardwood percentage.

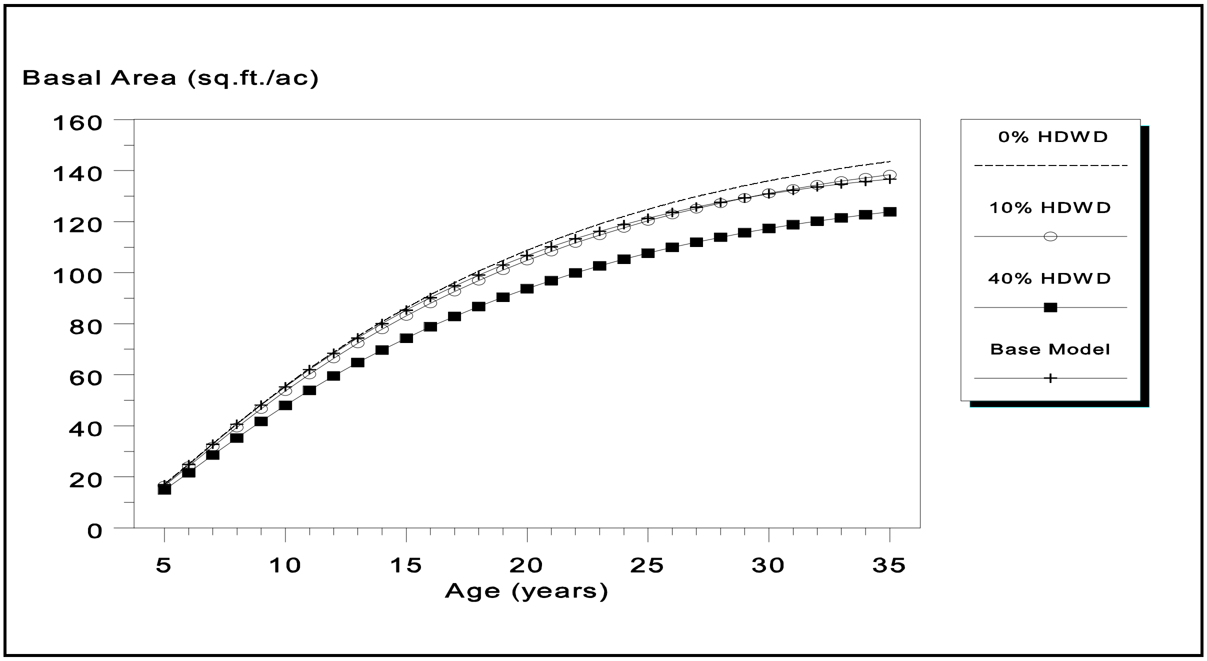

Figure 9. Basal area growth curves for an age five density of 500 trees per acre, a site index of 60 feet and various levels of hardwood competition in the Piedmont region.

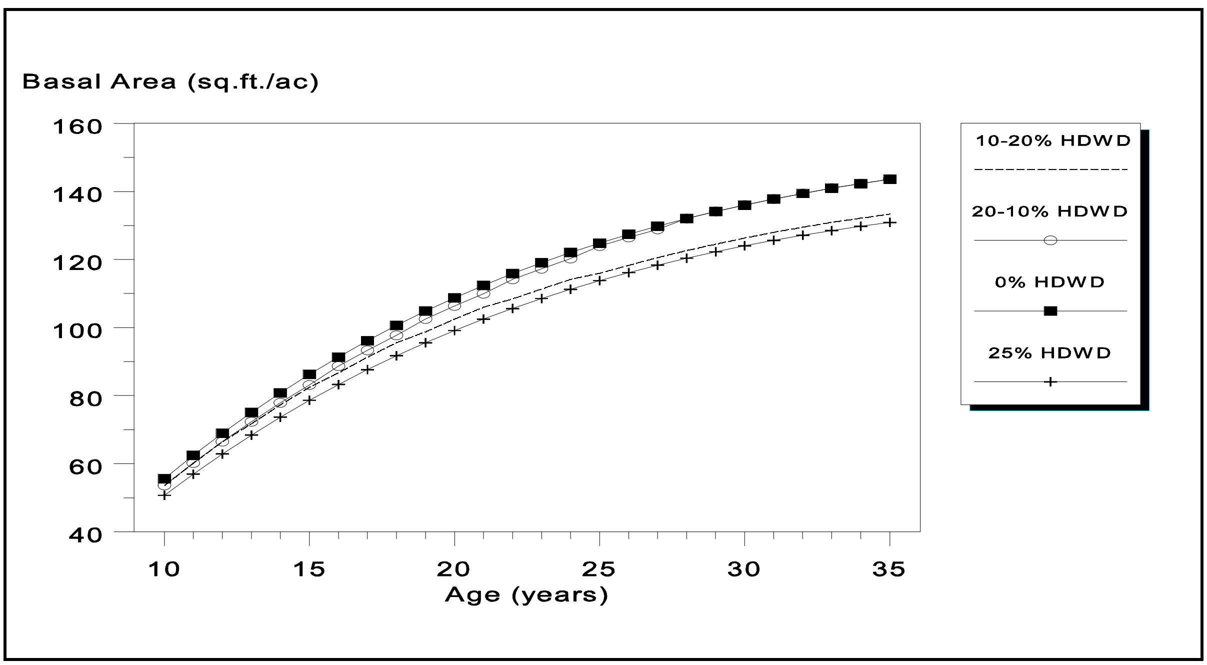

Figure 10. Basal area growth curves for an age five density of 500 trees per acre, a site index of 60 feet and changing levels of hardwood competition in the Piedmont region.

5.4 Per acre yield prediction

Whole stand yield prediction functions were developed for outside bark total volume, inside bark total volume, total green weight outside bark and total dry weight inside bark. As with basal area prediction, a conditional F-test revealed significant differences in yield prediction models among the physiographic regions. In fact, different model forms were required to best predict yield in the different regions. The model form for the Piedmont and Upper Coastal Plain regions is as follows:

$$\ln(Y) = b_0 + b_1 \ln(HD) + b_2 \ln(BA) + b_3 \frac{\ln(TPA)}{A} + b_4 \frac{\ln(HD)}{A} + b_5 \frac{\ln(BA)}{A} \tag{19}$$

where Y = per acre yield (TVOB, TVIB, GWOB, DWIB).

Parameter estimates and fit statistics for the Piedmont and Upper Coastal Plain yield models are shown in Table 17.

Table 17: Parameter estimates and fit statistics for the Piedmont and Upper Coastal Plain yield prediction equations.

Yield Unit	Parameter Estimates						Fit Statistics		
	b_0	b_1	b_2	b_3	b_4	b_5	n	R^2	$S_{y.x}$
TVOB	0.0	0.268552	1.368844	-7.466863	8.934524	3.553411	734	0.99	0.1489
TVIB	0.0	0.350394	1.263708	-8.608165	7.193937	6.309586	734	0.99	0.1484
GWOB	-3.818016	0.430179	1.276768	-8.088792	7.428472	5.554509	734	0.96	0.1481
DWIB	-4.987560	0.446433	1.348843	-7.757842	7.857337	4.222016	734	0.96	0.1501

Figures 11-13 show predicted growth of total, outside bark volume using the Piedmont and Upper Coastal Plain equation. Figure 11 shows volume curves by density. Figure 12 shows volume curves by site index and Figure 13 shows a volume growth curve with its associated mean annual increment.

The per acre yield prediction equation form for the Lower Coastal Plain region is as follows:

$$\ln(Y) = b_0 + b_1 \ln(TPA) + b_2 \ln(HD) + b_3 \ln(BA) + b_4 \frac{\ln(TPA)}{A} + b_5 \frac{\ln(BA)}{A} \tag{20}$$

Parameter estimates and fit statistics for the Lower Coastal Plain yield models are shown in Table 18.

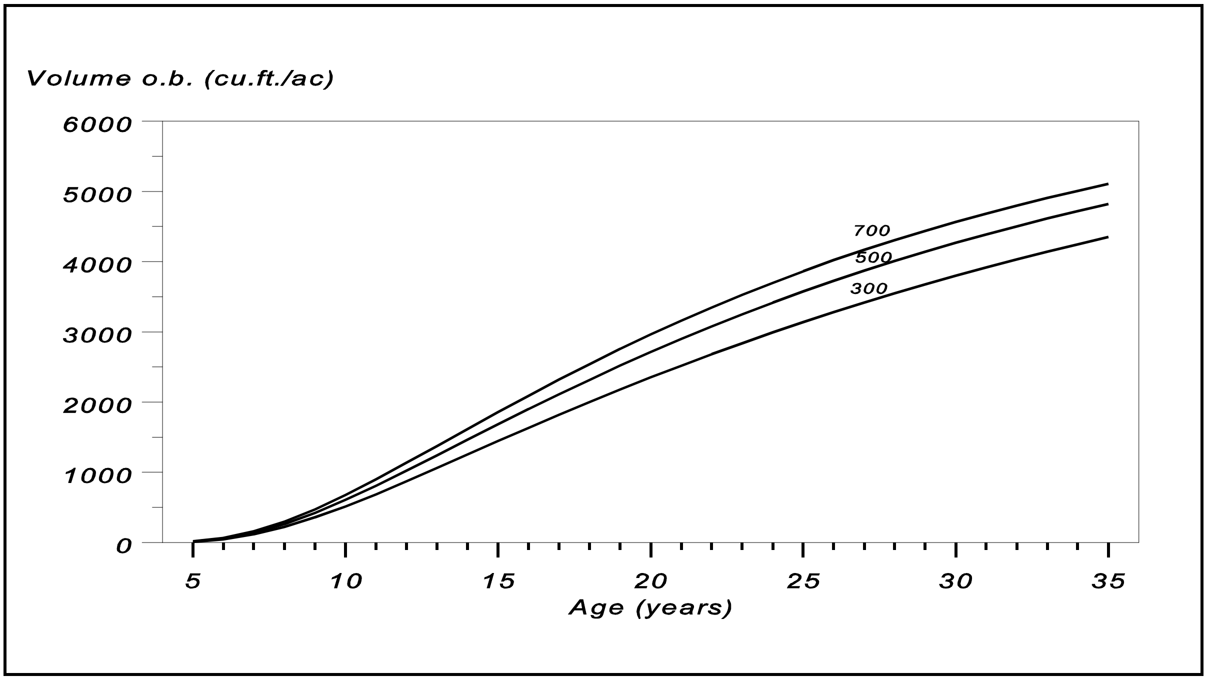

Figure 11. Total volume growth curves for a site index of 60 feet and age five densities of 300, 500 and 700 trees per acre in the Upper Coastal Plain and Piedmont regions.

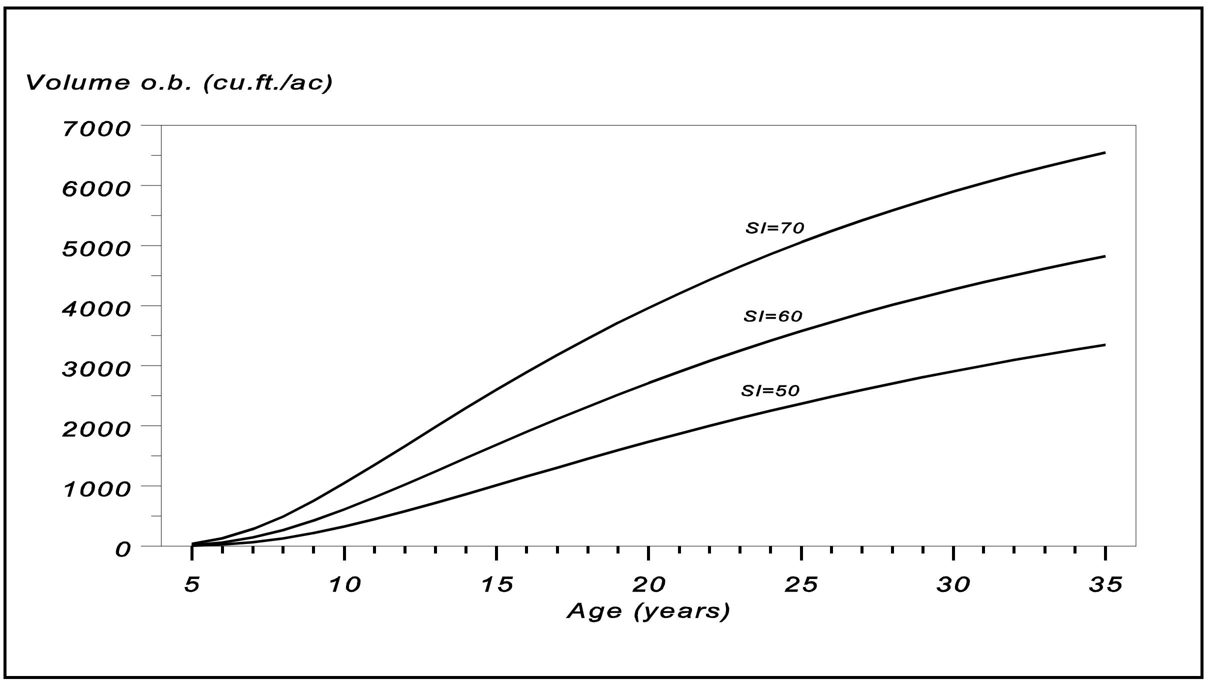

Figure 12. Total volume growth curves for an age five density of 500 trees per acre and site indices of 50, 60 and 70 feet in the Upper Coastal Plain and Piedmont regions.

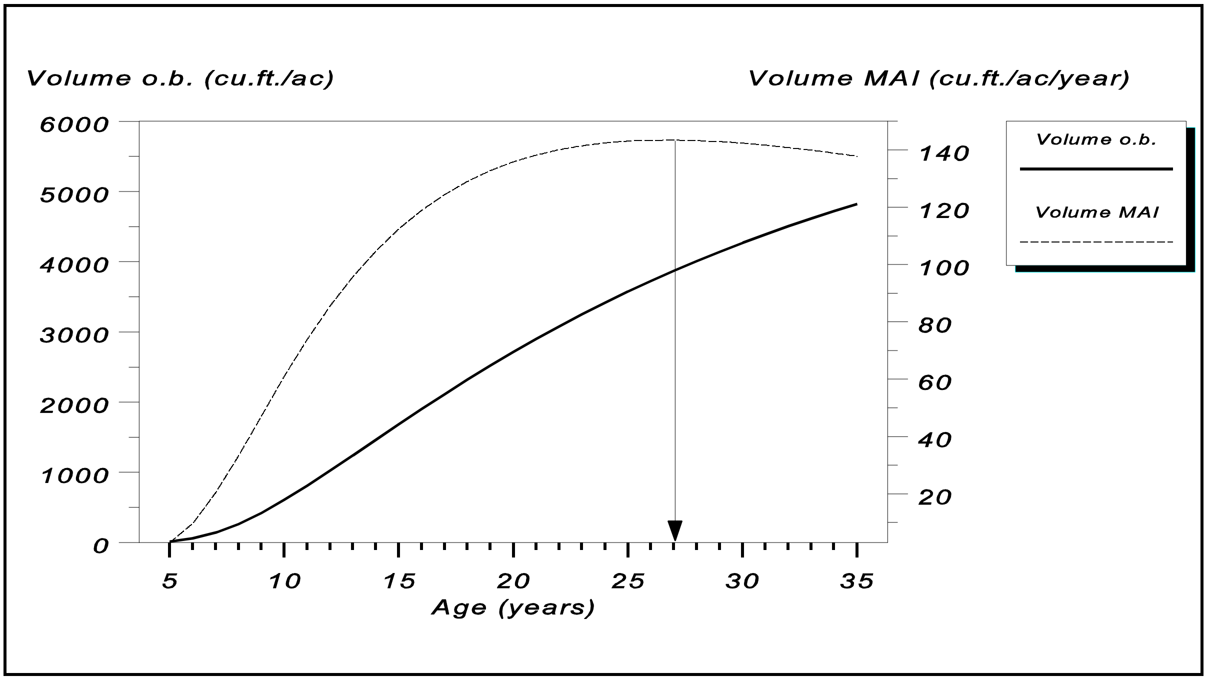

Figure 13. Total volume growth and its associated MAI for an age five density of 500 trees per acre and a site index of 60 feet in the Upper Coastal Plain and Piedmont regions.

Table 18: Parameter estimates and fit statistics for the Lower Coastal Plain yield prediction equations.

Yield Unit	Parameter Estimates						Fit Statistics		
	b_0	b_1	b_2	b_3	b_4	b_5	n	R^2	$S_{y.x}$
TVOB	-1.520877	0.200680	1.207586	0.703405	-5.139064	6.744164	400	0.99	0.0585
TVIB	-2.088857	0.177587	1.303770	0.726950	-5.091474	6.676532	400	0.99	0.0592
GWOB	-5.175922	0.198424	1.232028	0.705769	-5.129853	6.731477	400	0.99	0.0588
DWIB	-6.332502	0.145815	1.296629	0.814967	-4.660198	5.383589	400	0.99	0.0617

Figures 14-16 show predicted growth of total, outside bark volume using the Lower Coastal Plain equation. Figure 14 shows volume curves by density. Figure 15 shows volume curves by site index and Figure 16 shows a volume growth curve with its associated mean annual increment.

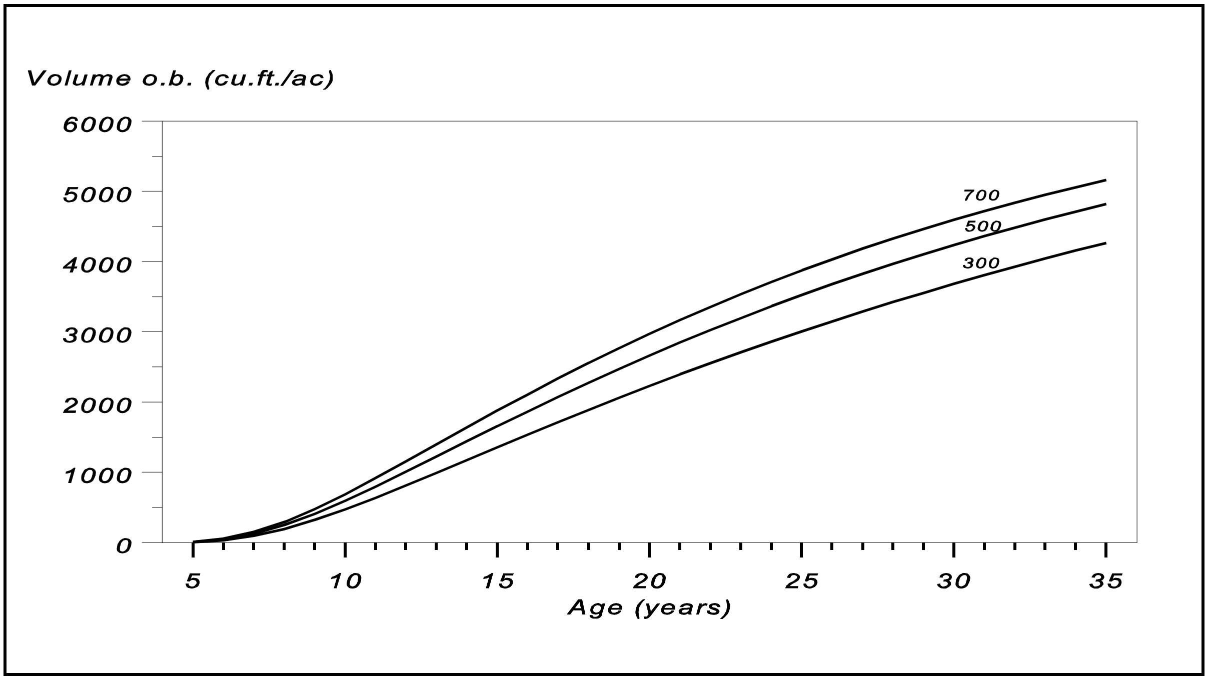

Figure 14. Total volume growth curves for a site index of 60 feet and age five densities of 300, 500 and 700 trees per acre in the Lower Coastal Plain region.

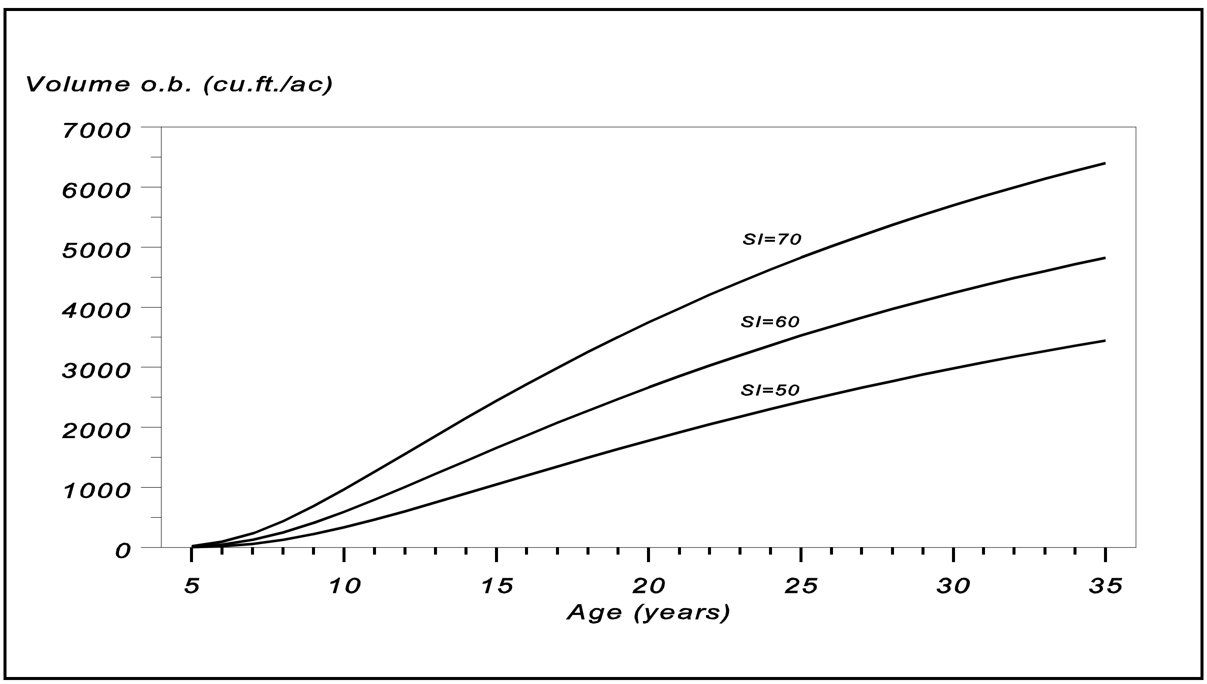

Figure 15. Total volume growth curves for an age five density of 500 trees per acre and site indices of 50, 60 and 70 feet in the Lower Coastal Plain region.

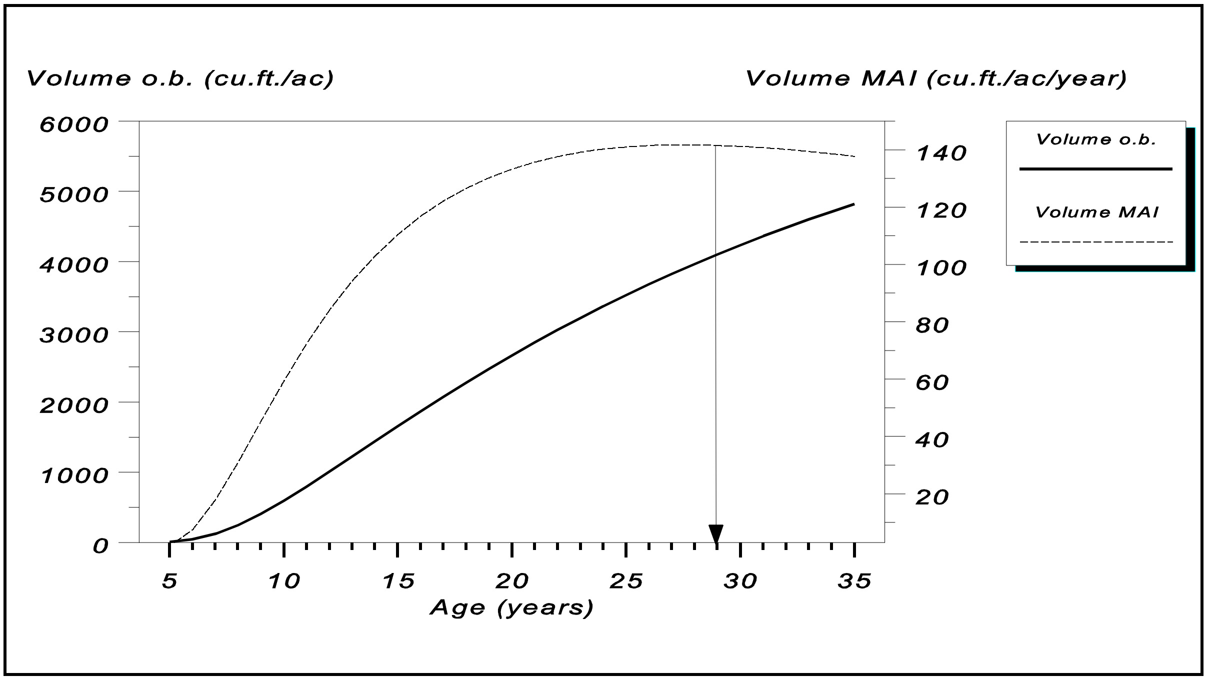

Figure 16. Total volume growth and its associated MAI for an age five density of 500 trees per acre and a site index of 60 feet in the Lower Coastal Plain region.

5.5 Yield breakdown function

Amateis et.al. (1986) developed a method to proportion total yield into product classes defined by a top diameter (*t*) and a DBH threshold limit (*d*). The PMRC loblolly data were used to develop yield breakdown functions for TVOB, TVIB, GWOB and DWIB. The model form is as follows:

$$Y_m = Y \exp\left[b_1(t/D_q)^{b_2} + b_3 TPA^{b_4}\ (d/D_q)^{b_5}\right] \qquad \textbf{(21)}$$

where Y_m = merchantable yield per acre for trees *d* inches DBH and above to a top diameter of *t* inches outside bark,

Y = total yield per acre (TVOB, TVIB, GWOB, DWIB).

A conditional F-test revealed significant differences among regions for the product yield allocation equation. Therefore, separate parameter estimates were obtained for the combined Upper Coastal Plain and Piedmont datasets and for the Lower Coastal Plain dataset. Parameter estimates and fit statistics are shown in Tables 19 and 20 for the Upper Coastal Plain and Piedmont, and for the Lower Coastal Plain, respectively.

Table 19: Parameter estimates and fit statistics for the Upper Coastal Plain and Piedmont yield breakdown equations.

Parameter Estimates and Fit Statistics	Yield Unit			
	TVOB	TVIB	GWOB	DWIB
b_1	-0.982648	-1.036792	-1.007482	-0.934936
b_2	3.991140	3.900677	3.931373	4.111618
b_3	-0.748261	-0.511939	-0.518057	-0.590269
b_4	-0.111206	-0.046007	-0.048385	-0.065355
b_5	5.784780	5.640610	5.660573	5.596179
n	6105	6105	6105	6105
R^2	0.96	0.97	0.96	0.97
$S_{y.x}$	232.4	188.0	6.2	2.6

Table 20: Parameter estimates and fit statistics for the Lower Coastal Plain yield breakdown equations.

Parameter Estimates and Fit Statistics	Yield Unit			
	TVOB	TVIB	GWOB	DWIB
b_1	-1.034486	-1.105225	-1.064132	-0.963185
b_2	3.940848	3.878664	3.818683	4.054202
b_3	-5.062955	-4.459271	-5.048319	-4.540672
b_4	-0.422892	-0.404057	-0.422117	-0.406561
b_5	6.004646	5.984225	5.991728	5.962867
n	5140	5140	5140	5140
R^2	0.98	0.98	0.98	0.98
$S_{y.x}$	242.9	199.5	6.8	3.0

Predicted product yields over time are shown in Figure 17 for the Upper Coastal Plain and Piedmont and in Figure 18 for the Lower Coastal Plain.

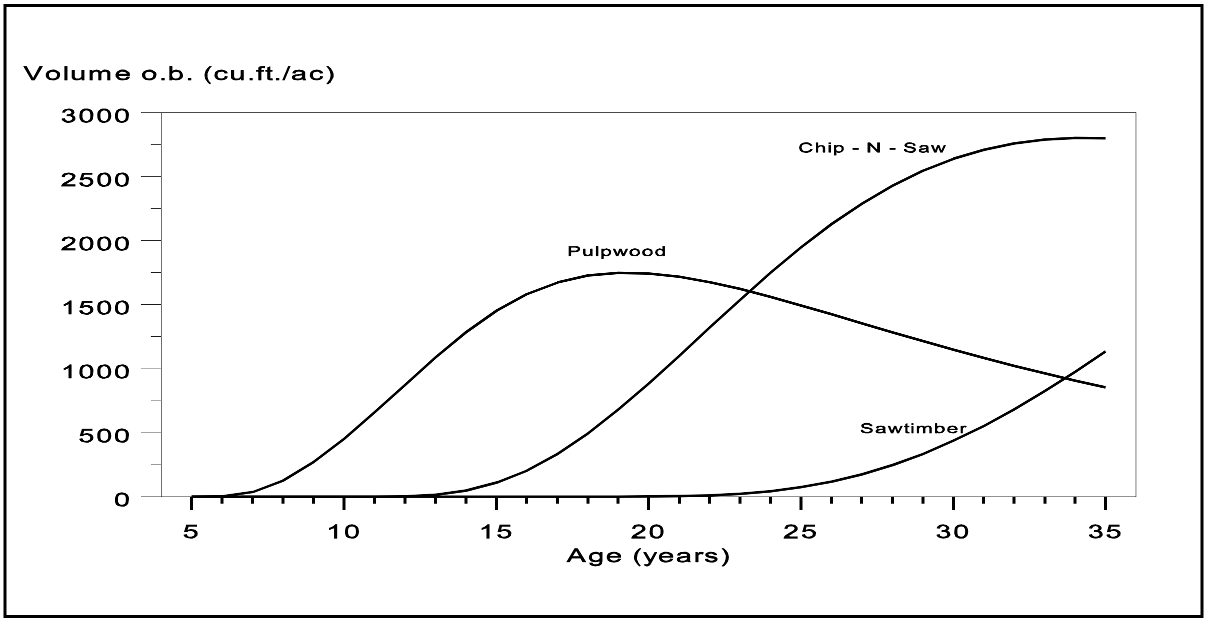

Figure 17. Product volume growth for an age five density of 500 trees per acre and a site index of 60 feet in the Upper Coastal Plain and Piedmont regions. Pulpwood consists of trees larger than 4.5" Dbh to a 2" top (o.b.); Chip-N-Saw consists of trees between 8.5" and 11.5" Dbh to a 4" top (o.b.); Sawtimber consists of trees larger than 11.5" to an 8" top (o.b.).

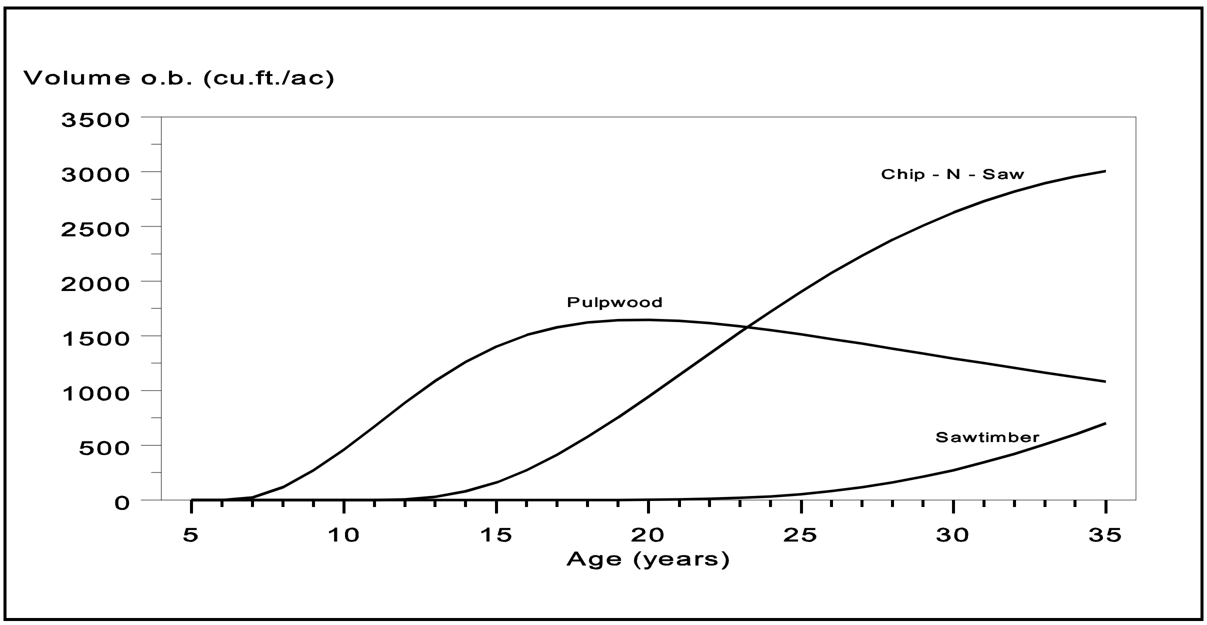

Figure 18. Product volume growth for an age five density of 500 trees per acre and a site index of 60 feet in the Lower Coastal Plain region. Pulpwood consists of trees larger than 4.5" Dbh to a 2" top (o.b.); Chip-N-Saw consists of trees between 8.5" and 11.5" Dbh to a 4" top (o.b.); Sawtimber consists of trees larger than 11.5" to an 8" top (o.b.).

5.6 Analysis of limiting stand density relationships

Clutter et.al. (1983) describe several stand density measures and their limiting relationships as stands grow older. The whole stand loblolly growth and yield models described above were analyzed in terms of stand density index and relative spacing. Stand density index is defined as the relationship between the number of trees per acre and average tree size. In fully stocked, even-aged stands, the relationship between the number of trees per acre and the quadratic mean Dbh should appear linear in logarithmic coordinates. This implies a limiting number of trees per acre for a given D_q. Reineke (1933) observed this relationship for a variety of species and determined the slope of the limiting line was approximately -1.6. Equations **(14)**, **(15)** and **(16)** were used to predict TPA and D_q for a site index of 60 feet, age five densities of 300 and 700 trees per acre and ages from 5 to 100 years. Figures 19 and 20 show the relationships for the combined Upper Coastal Plain and Piedmont regions. Figures 21 and 22 show the Lower Coastal Plain curves. The slope of the limiting relationships was determined by a regression of ln(TPA) as a function of ln(D_q) in the linear portion as indicated by the graphs.

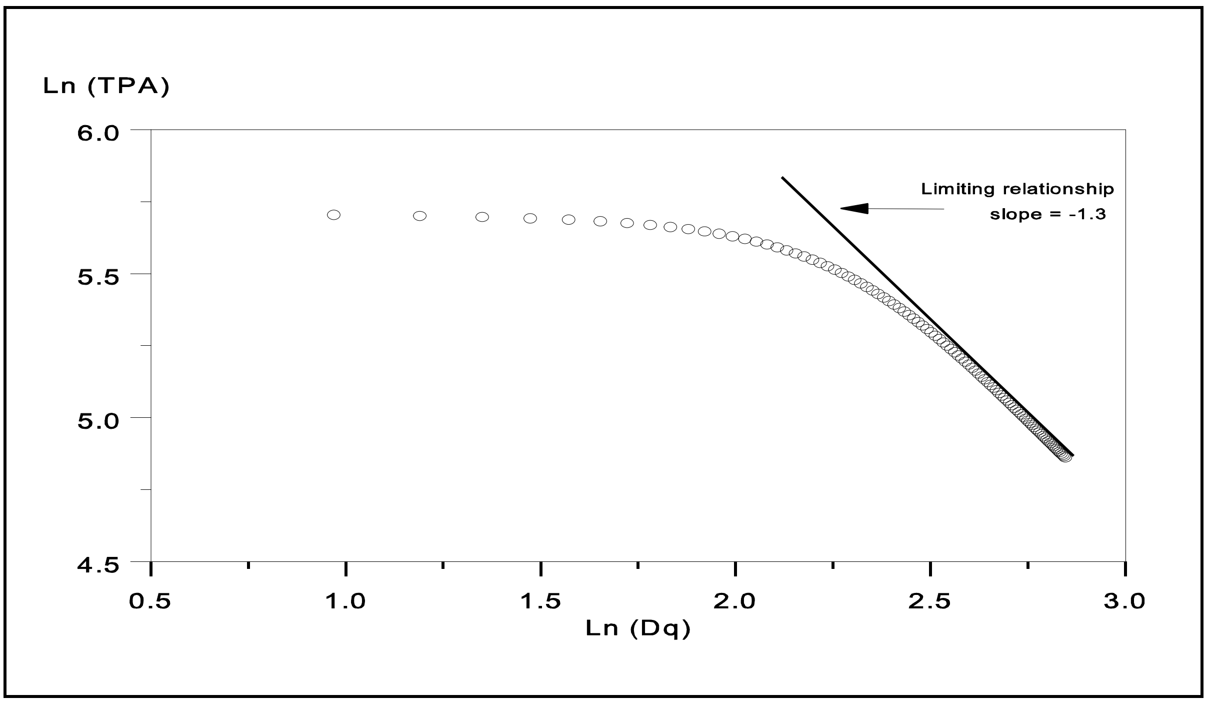

Figure 19. Natural log of TPA over natural log of D_q for a site index of 60 feet and an age five density of 300 trees per acre in the Upper Coastal Plain and Piedmont.

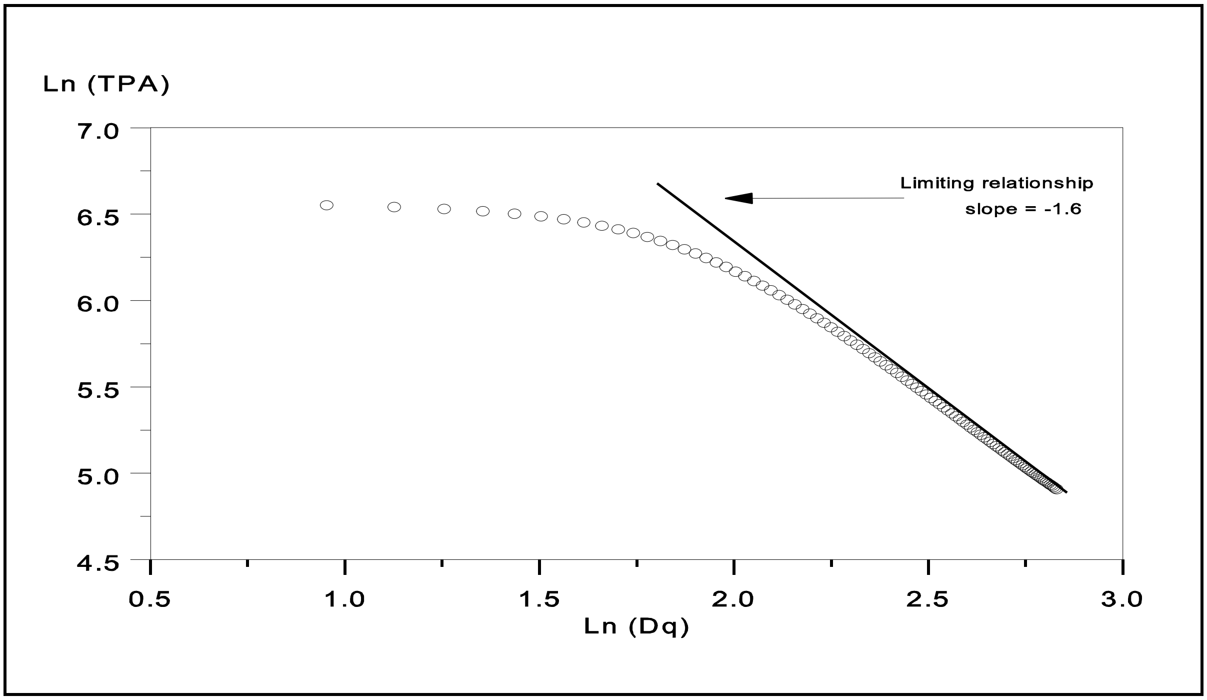

Figure 20. Natural log of TPA over natural log of D_q for a site index of 60 feet and an age five density of 700 trees per acre in the Upper Coastal Plain and Piedmont.

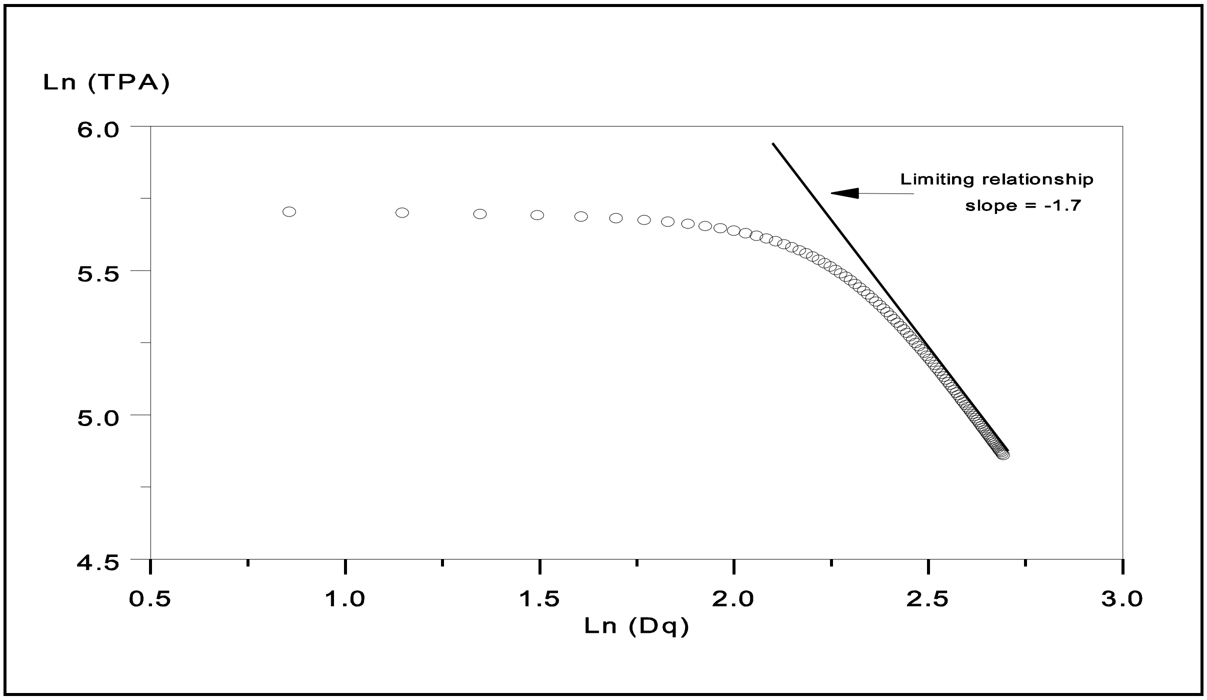

Figure 21. Natural log of TPA over natural log of D_q for a site index of 60 feet and an age five density of 300 trees per acre in the Lower Coastal Plain.

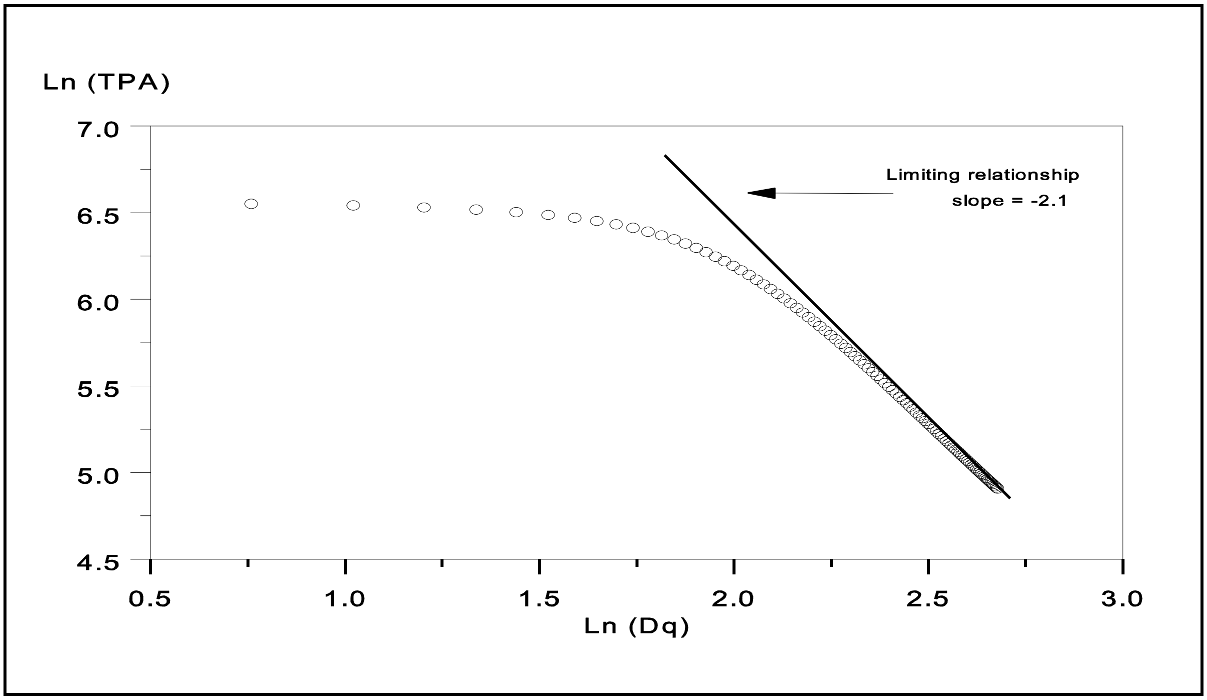

Figure 22. Natural log of TPA over natural log of D_q for a site index of 60 feet and an age five density of 700 trees per acre in the Lower Coastal Plain.

Relative spacing is defined as the ratio between the average distance between trees and the average dominant height of a stand. Clutter et.al. (1983) point out that regardless of site quality, stands of a given species seem to approach a common, minimum relative spacing as they grow older. Figures 23 and 24 show the development of relative spacing over age as predicted using equations **(14)** and **(15)** for all physiographic regions. These graphs indicate that the loblolly models generally adhere to the aforementioned premise. The relative spacing curves for site indices of 60 and 75 feet seem to converge to the same minimum relative spacing level. The different densities also seem to approach a common minimum relative spacing in the range of ages investigated.

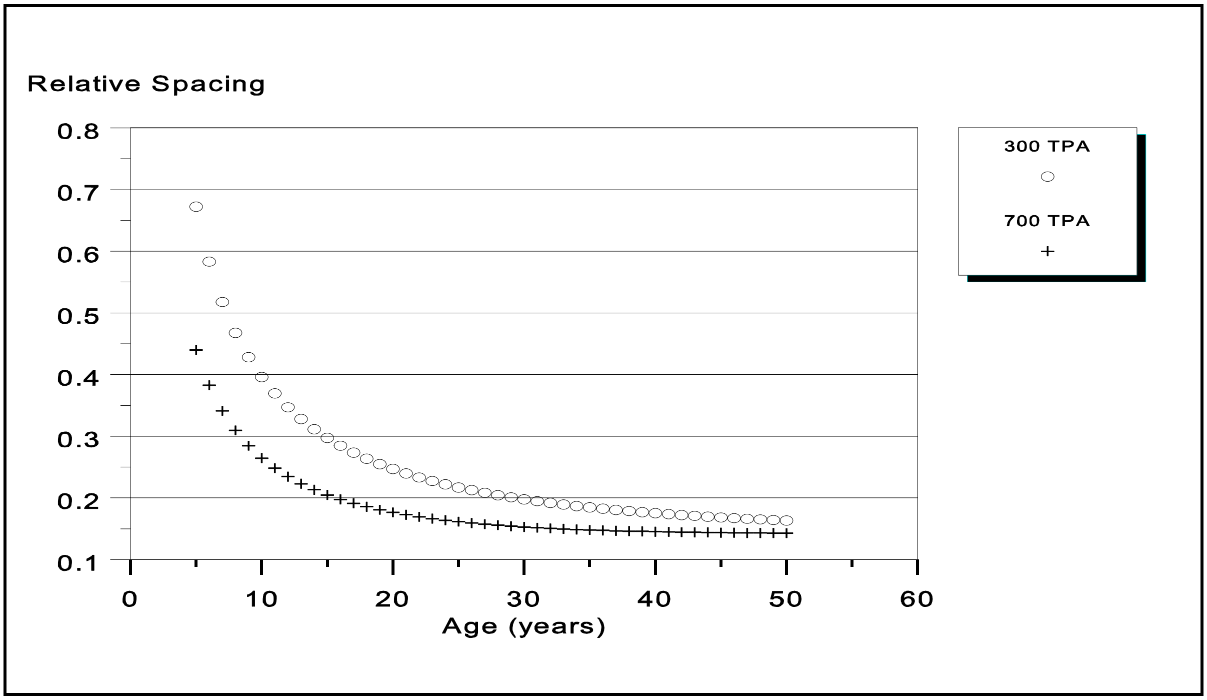

Figure 23. Relative spacing over age for a site index of 60 feet and age five densities of 300 and 700 trees per acre in all regions.

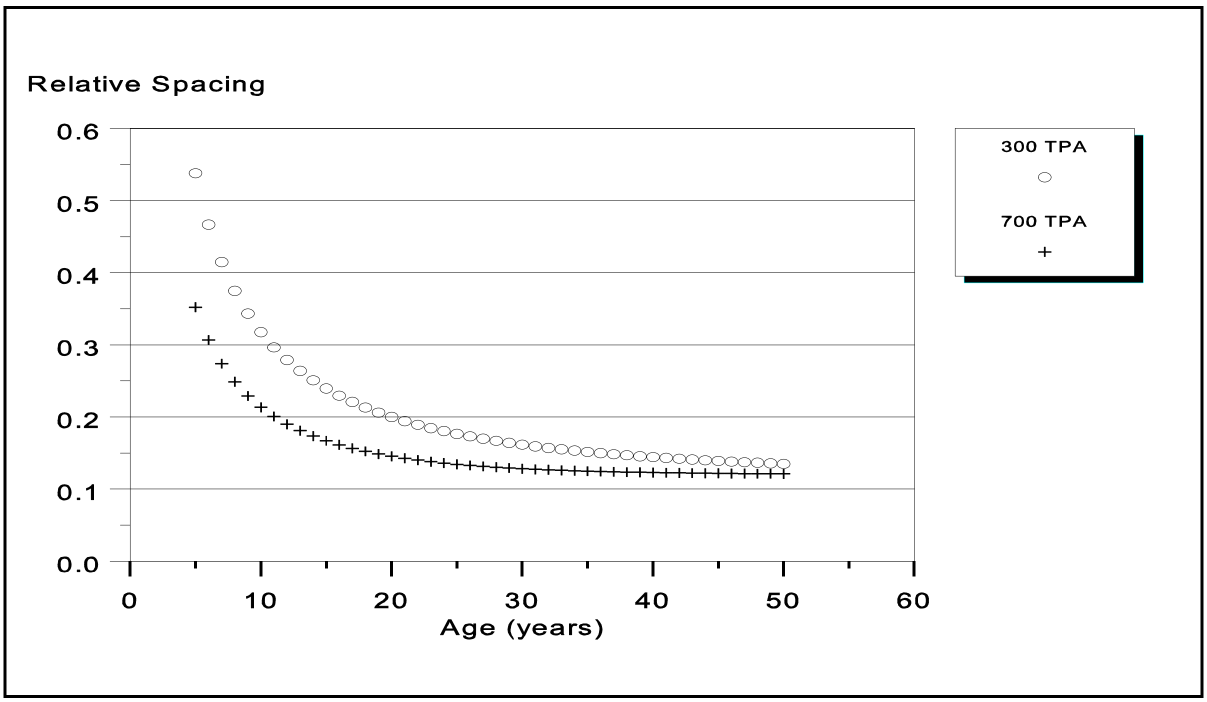

Figure 24. Relative spacing over age for a site index of 75 feet and age five densities of 300 and 700 trees per acre in all regions.

6 IMPLICIT YIELD PREDICTION MODELS

6.1 Percentile prediction and parameter recovery using a Weibull PDF

The utility of the three-parameter Weibull probability distribution function for modelling southern pine diameter distributions has been well established. The method was first introduced by Bailey and Dell (1973). Several methods are available for relating observed or predicted stand characteristics to Weibull parameters. Borders et.al. (1990) presented a parameter recovery method which uses estimates of the 0th, 25th, 50th and 95th Dbh distribution percentiles to obtain Weibull parameter estimates. This method ensures that the resulting predicted diameter distribution matches the quadratic mean Dbh implied by whole stand measurements or estimates of trees per acre and basal area per acre.

Models were developed to predict diameter distribution percentiles for the PMRC loblolly data. The following model form achieves reasonable goodness-of-fit while preventing illogical crossover of adjacent percentiles:

$$\ln(P_x) = a_0 + a_1 \ln\left(\frac{BA}{TPA}\right) \qquad \textbf{(22)}$$

where P_x = diameter distribution percentile (x = 0, 25, 50, 95).
Separate parameter estimates were required for the combined Upper Coastal Plain and Piedmont data and for the Lower Coastal Plain data. Parameter estimates and fit statistics are shown in Tables 21 and 22.

A two-sample Komolgorov-Schmirnoff test (Sokal and Rohlf, 1981) was used to evaluate the accuracy of predicted diameter distributions. Significant differences between predicted and observed distributions were detected in 30 of 1322 cases at the a = 0.05 level.

Table 21: Parameter estimates and fit statistics for the Upper Coastal Plain and Piedmont percentile prediction equations.

Parameter Estimates and Fit Statistics	Percentile			
	P_0	P_{25}	P_{50}	P_{95}
a_0	2.374894	2.586318	2.714412	2.869722
a_1	0.976577	0.503910	0.485314	0.469809
n	740	740	740	740
R^2	0.57	0.97	0.98	0.92
$S_{y.x}$	0.37	0.04	0.03	0.06

Table 22: Parameter estimates and fit statistics for the Lower Coastal Plain percentile prediction equations.

Parameter Estimates and Fit Statistics	Percentile			
	P_0	P_{25}	P_{50}	P_{95}
a_0	2.168021	2.547423	2.653169	2.861802
a_1	0.773026	0.574370	0.513997	0.463918
n	580	580	580	580
R^2	0.61	0.80	0.87	0.92
$S_{y.x}$	0.29	0.13	0.09	0.06

Predicted growth of percentiles over time is shown in Figure 25 for the Upper Coastal Plain and Piedmont regions and in Figure 26 for the Lower Coastal Plain region.

The effect of hardwood competition on the pine diameter distribution was investigated using the Piedmont data. If equation **(18)** is used to predict basal area, taking into account the percent hardwood basal area, and equation **(22)** is used to predict diameter distribution percentiles, it was found that additional consideration must be given to the effect of hardwoods on the pine diameter distribution percentiles. To accomplish this, percentile prediction equations were fit to the PMRC Piedmont loblolly data using the percent hardwood basal area (PHWD). The hardwood variable was found to be significant in predicting the 25th and 95th percentiles. The full set of percentile prediction equations for the Piedmont region follows:

$$\ln(P_0) = 2.332760 + 0.962171 \ln\left(\frac{BA}{TPA}\right)$$

$$\ln(P_{25}) = 2.583306 + 0.515691 \ln\left(\frac{BA}{TPA}\right) - 0.0061 \; PHWD$$

$$\ln(P_{50}) = 2.720549 + 0.488296 \ln\left(\frac{BA}{TPA}\right)$$

$$\ln(P_{95}) = 2.898946 + 0.458079 \ln\left(\frac{BA}{TPA}\right) + 0.013259 \; PHWD$$

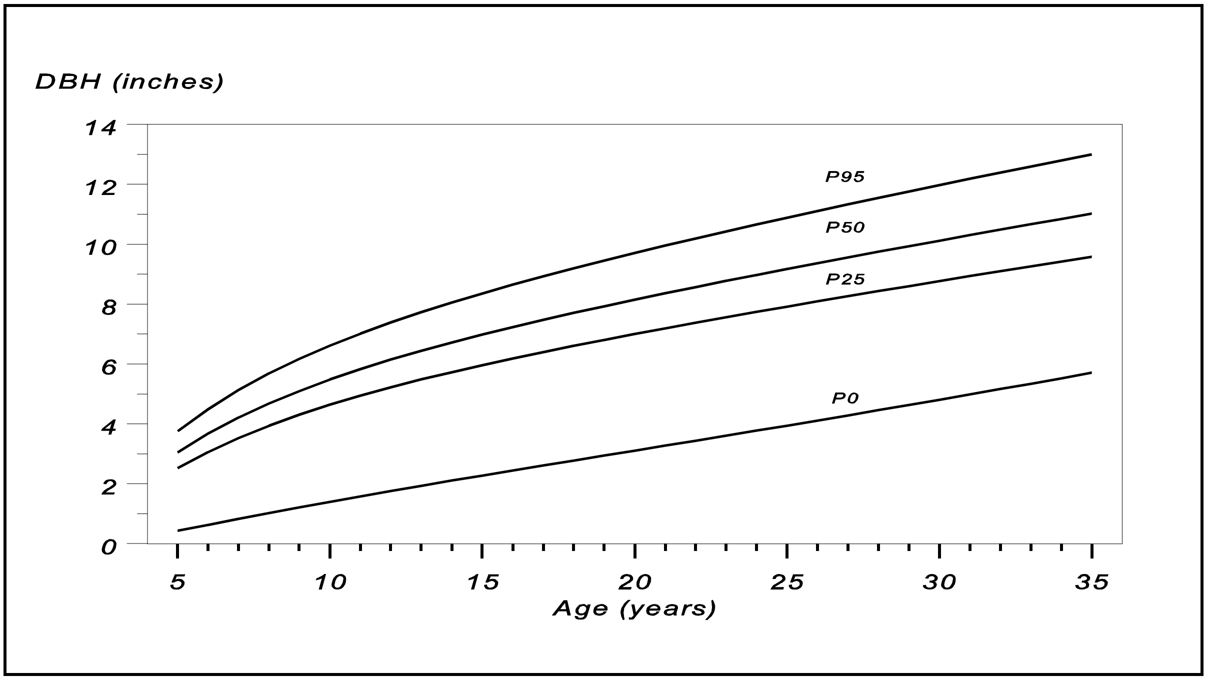

Figure 25. Percentile growth curves for an age five density of 500 trees per acre and a site index of 60 feet in the Upper Coastal Plain and Piedmont regions.

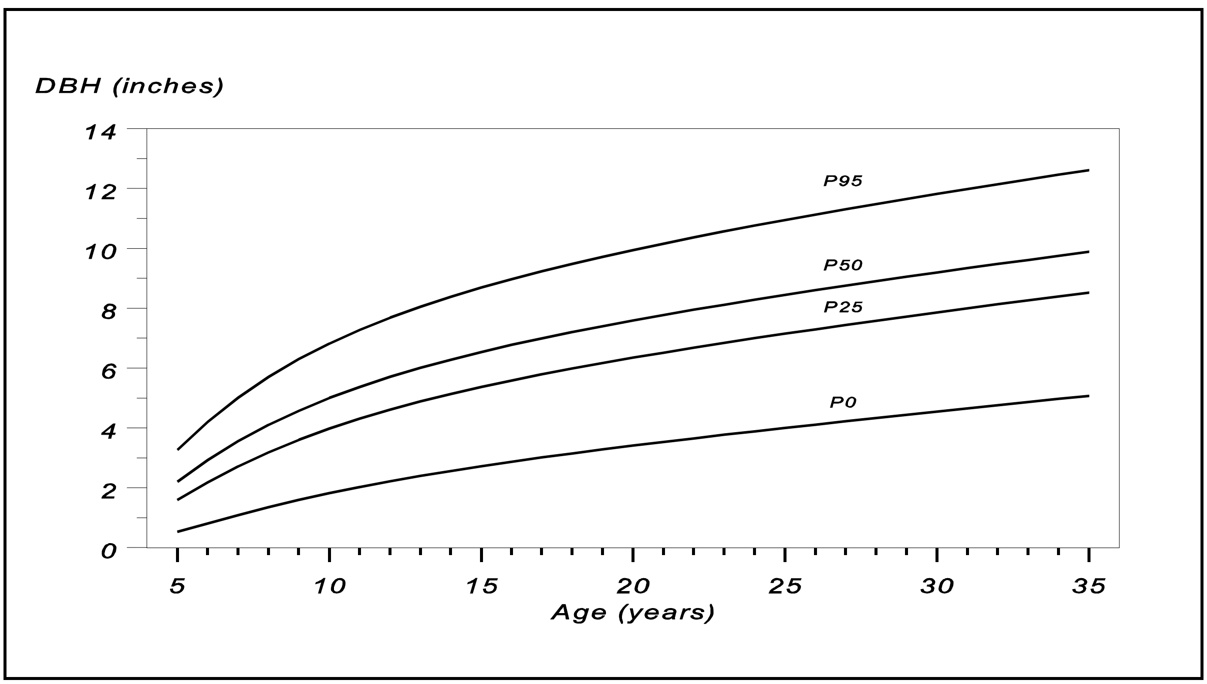

Figure 26. Percentile growth curves for an age five density of 500 trees per acre and a site index of 60 feet in the Lower Coastal Plain region.

Predicted diameter distributions with various levels of hardwood competition are shown in Figure 27.

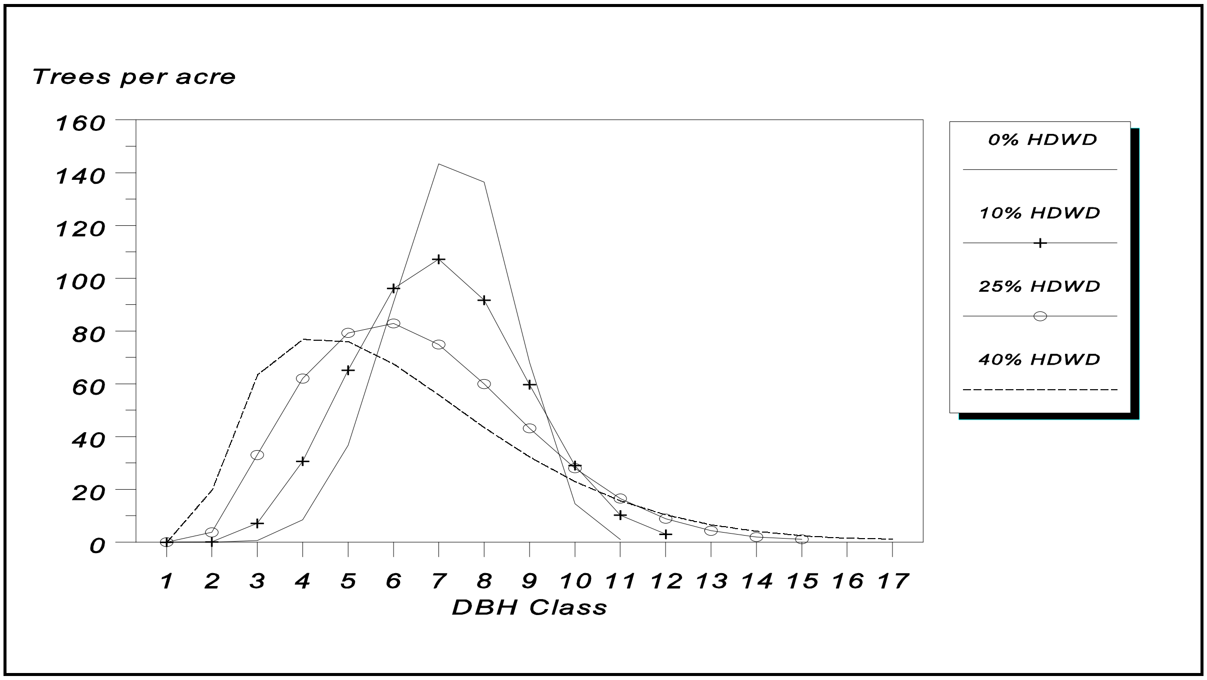

Figure 27. Diameter distributions with various levels of hardwood competition for a stand of age 25 years, 300 trees per acre and a site index of 60 feet in the Piedmont region.

The impact of increasing levels of hardwood basal area on the pine stand table is to shift the modal Dbh class to the left and the largest Dbh classes to the right. Thus, the pine stand table becomes more positively skewed as the amount of hardwood basal area increases. Knowe (1992) reported similar results based on the initial measurement of hardwoods on the PMRC growth and yield plots. This result does not appear entirely logical. As such, further study of hardwood competition and its effect on pine stand tables will continue to be researched. Models will be modified if and when new information becomes available.

6.2 Stand table projection model

When an existing stand table is available from an inventory or from a diameter distribution prediction system, the stand table can be projected using a method developed by Clutter and Allison (1974) and modified by Pienaar and Harrison (1988). The procedure involves projecting the growth of individual trees or DBH class midpoints in relation to their relative size according to the following assumptions:

- Trees of below average size will become smaller relative to the average size with increasing stand age,
- Trees of above average size will become larger relative to the average size with increasing stand age,
- For a given projection interval length, the change in relative size will decrease as initial age increases.

Pienaar and Harrison (1988) developed the following relative size projection equation which conforms to these assumptions:

$$b_{2i} = \overline{b_2}\left(\frac{b_{1i}}{\overline{b_1}}\right)^{\left(\frac{A_2}{A_1}\right)^{ß}} \qquad \textbf{(23)}$$

where $\bar{b}_1$ = average basal area at time 1,

$\bar{b}_2$ = average basal area at time 2,

b_{1i} = basal area of tree or Dbh class midpoint i at time 1,

b_{2i} = basal area of tree or Dbh class midpoint i at time 2,

ß = parameter estimated from data.

Borders et.al. (1990) fit equation **(23)** to the PMRC loblolly data using individual trees. Since additional measurements of the same trees were carried out, the model was refit. A conditional F-test indicated that separate models were required for the combined Upper Coastal Plain and Piedmont regions and for the Lower Coastal Plain region. Parameter estimates and fit statistics are shown in Table 23.

Table 23: Parameter estimates and fit statistics for the relative size projection equations.

Parameter Estimates and Fit Statistics	Region	
	Upper Coastal Plain + Piedmont	Lower Coastal Plain
ß	-0.2277	-0.0525
n	6371	828
R^2	0.87	0.92
$S_{y.x}$	0.1931	0.1482

Using equation **(23)** and the coefficient estimates from Table 23, stand tables were projected for 614 growth intervals constructed from the PMRC loblolly plots. Projected stand tables were compared to observed stand tables using a two-sample Komolgorov-Schmirnoff test. Projected stand tables were significantly different from observed stand tables in 3 of 614 cases at the a=0.05 level.

It should be noted that the coefficients for relative size projection for both regions are negative. This implies that trees starting smaller than average size as well as trees starting larger than average size will get closer to the average size over time. This contradicts one of the postulates put forth for using this modelling approach in even-aged stands. However, the changes in relative size are small within the 5 to 10 year projections and resulting stand tables are logical as indicated by the two-sample KS tests. When projection periods of greater than 15 years are used, it may be necessary to use the Weibull diameter distribution recovery algorithm to ensure reliable results.

6.3 Height / diameter function

A function to predict average height by Dbh class is required to obtain stock tables from stand tables. The following model form was developed by Pienaar et.al. (1988):

$$H_i = HD \; a_1 \left[1 - a_2 \exp\left(-a_3 \frac{DBH_i}{D_q} \right) \right] \qquad \textbf{(24)}$$

Diameter and height measurements for individual trees were used to fit this equation to the PMRC loblolly data. A conditional F-test indicated a significant difference in average height prediction for the combined Upper Coastal Plain and Piedmont regions and for the Lower Coastal Plain region. Parameter estimates and fit statistics are shown in Table 24.

Table 24: Parameter estimates and fit statistics for the average height prediction equation.

Parameter Estimates and Fit Statistics	Region	
	Upper Coastal Plain + Piedmont	Lower Coastal Plain
b_1	1.179240	1.185552
b_2	0.878092	0.949316
b_3	1.618723	1.710774
n	8873	1820
R^2	0.91	0.91
$S_{y.x}$	3.48	3.77

7 MODEL COMPARISONS

The growth and yield functions described above are of similar form to the 1990 version of the PMRC loblolly model. In order to assess the differences in the two model versions, predicted height, survival, basal area and total volume from the two model versions were plotted over age. Figures 28-31 show the results for a site index of 60 feet and an age five density of 500 trees per acre in the Lower Coastal Plain.

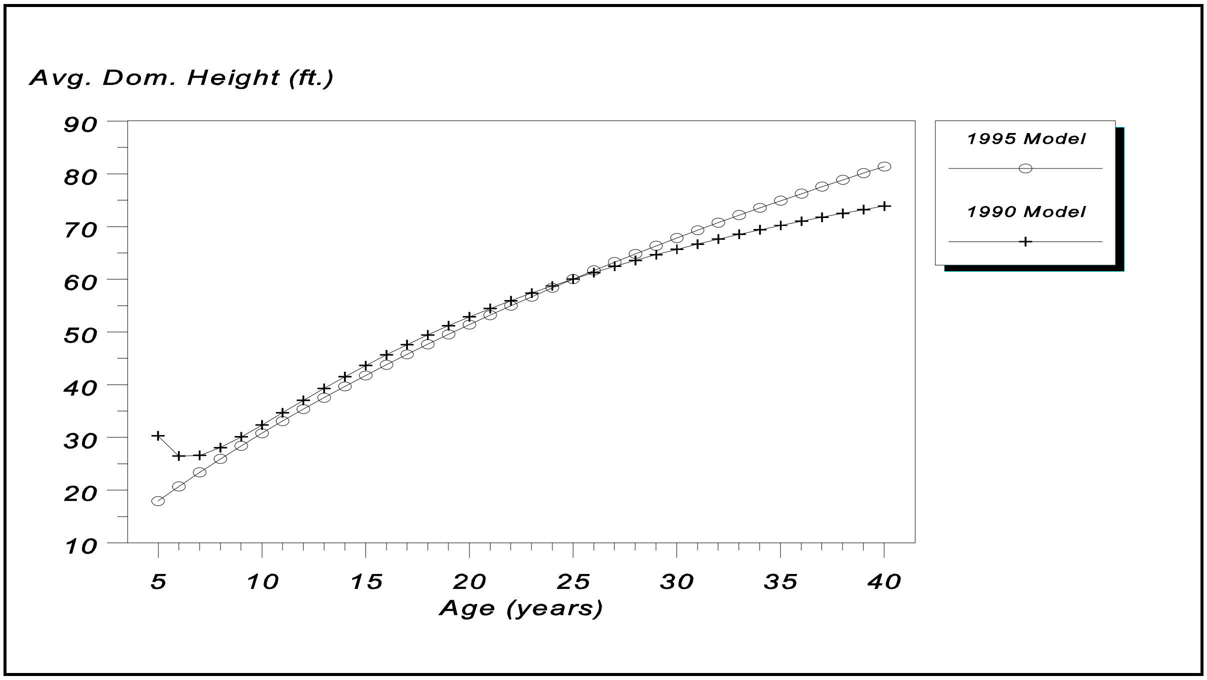

Figure 28. Average dominant height over age for a site index of 60 feet in the Lower Coastal Plain region.

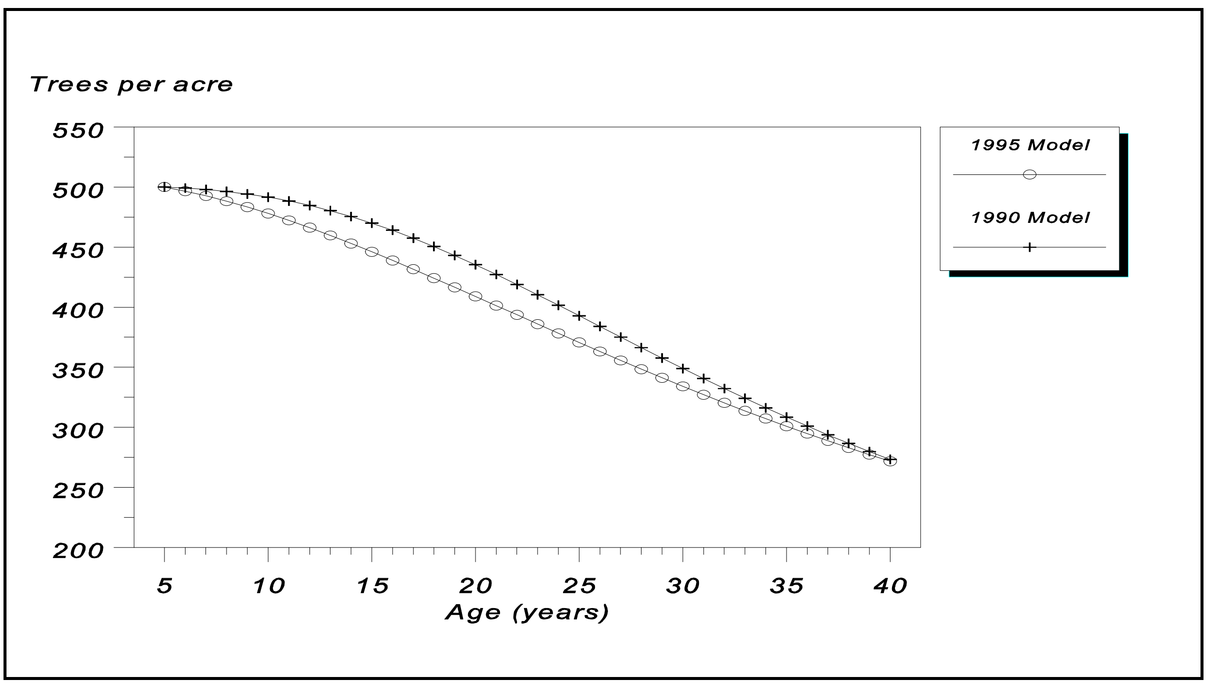

Figure 29. Trees per acre over age for a site index of 60 feet and an age five density of 500 trees per acre in the Lower Coastal Plain region.

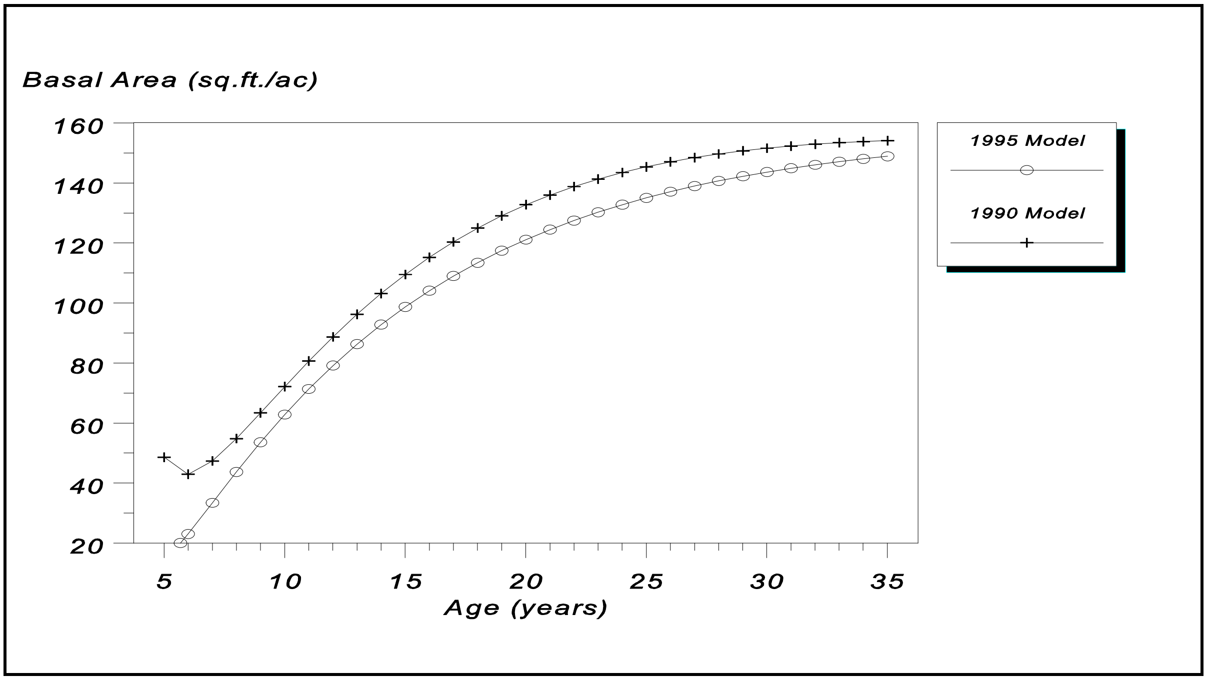

Figure 30. Basal area per acre over age for a site index of 60 feet and an age five density of 500 trees per acre in the Lower Coastal Plain region.

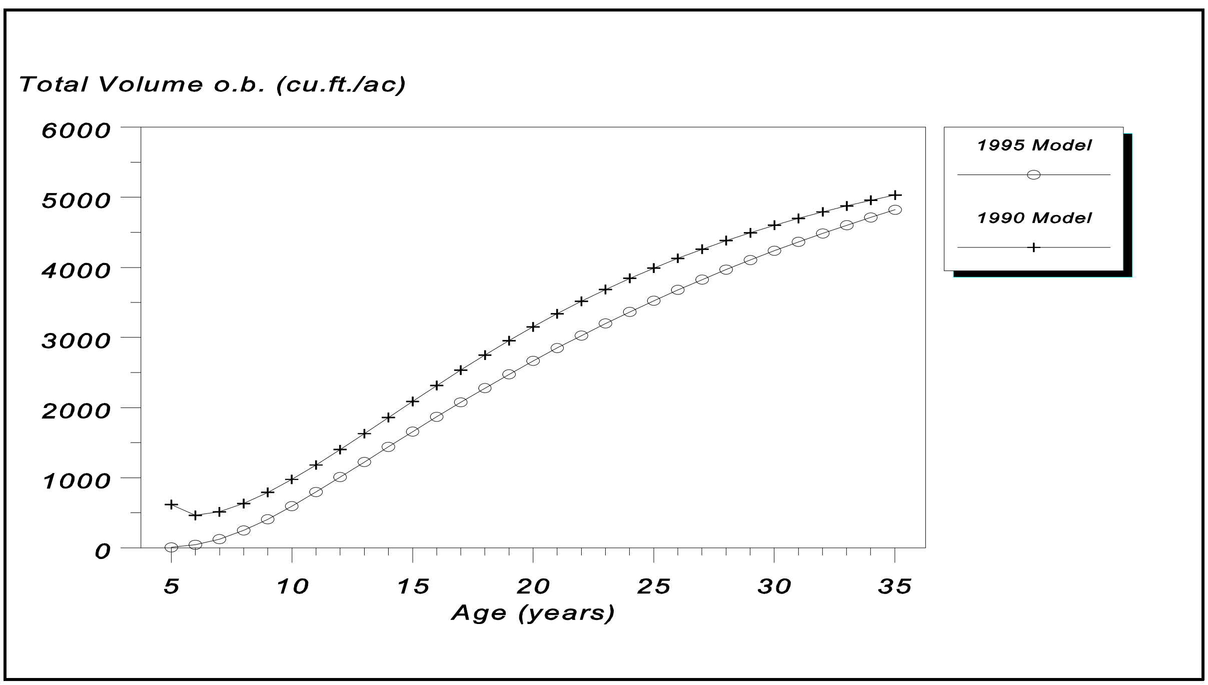

Figure 31. Total volume (o.b.) over age for a site index of 60 feet and an age five density of 500 trees per acre in the Lower Coastal Plain region.

Analysis of Figures 28-31 indicates that the 1990 model predicts a lower height, a higher number of trees per acre, more basal area per acre and a higher yield than the current model. These figures are somewhat misleading, mainly because of the differences in height growth and survival development. The 1990 survival function is of a similar form to equation **(15)** in the current version, except that the 1990 version uses dominant height instead of age. If we assume that the mortality rate increases with increasing height growth rate, as is indicated by the PMRC loblolly data, then the rate of mortality predicted using the 1990 model will be less because of the decreased predicted height growth.

Another aspect of the different height growth patterns displayed in Figure 28 makes the comparisons questionable. If we assume that the stands that are compared have the same site index, that would indicate that they had a different height at the initial age. It seems more reasonable to compare two stands that had the same height and number of stems per acre at a given age. Figures 32-35 represent such comparisons. In this case, the models were compared for stands which were approximately 30 feet tall and had 500 trees per acre at age 10 years. This resulted in a site index of 60 feet using the current model and a site index of 56 feet using the 1990 version.

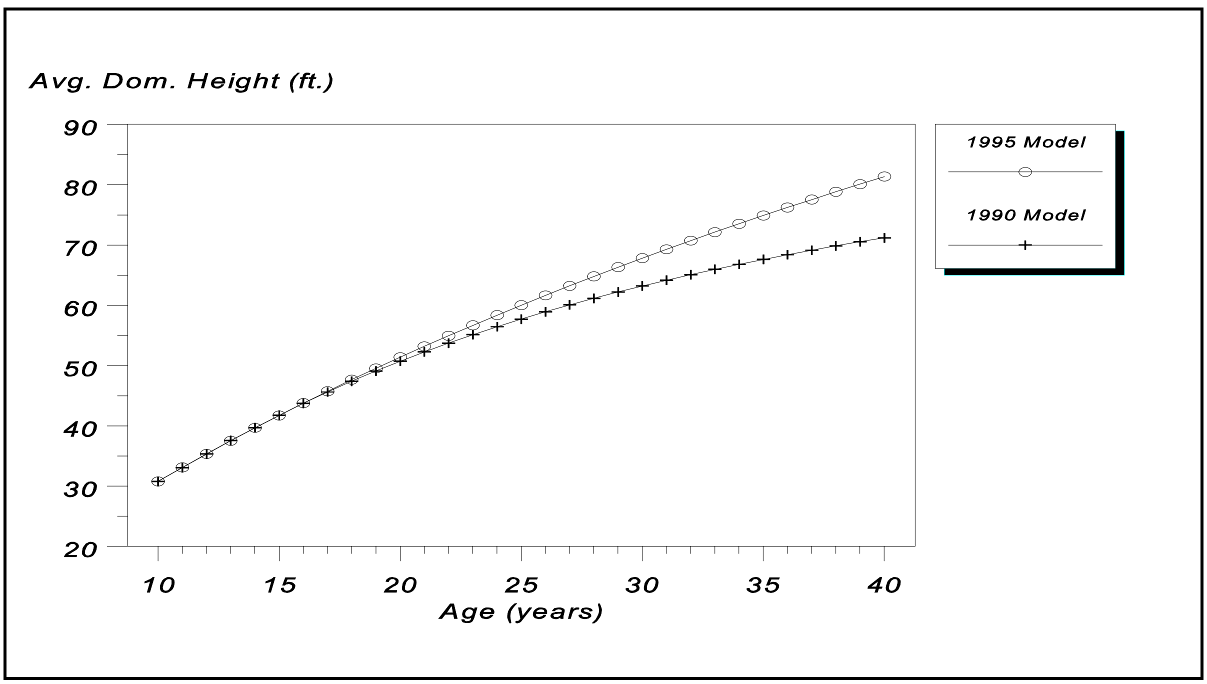

Figure 32. Dominant height over age for a height of 30 feet and a density of 500 trees per acre at age 10 in the Lower Coastal Plain region.

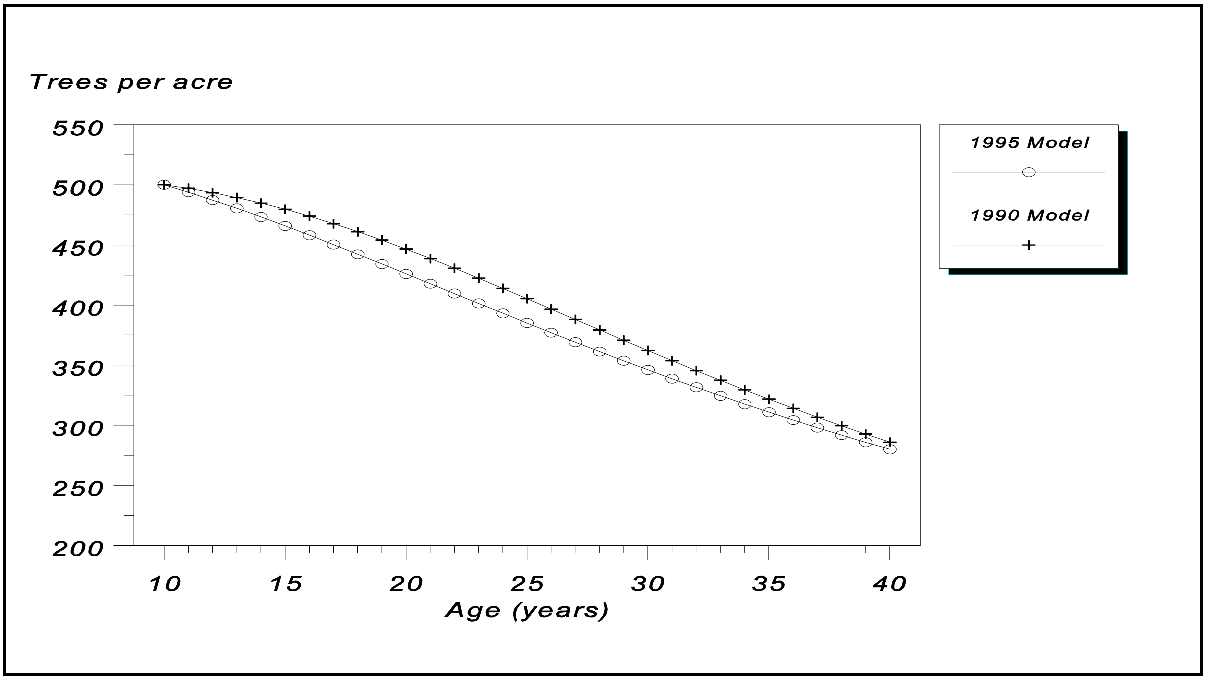

Figure 33. Trees per acre over age for a height of 30 feet and a density of 500 trees per acre at age 10 in the Lower Coastal Plain region.

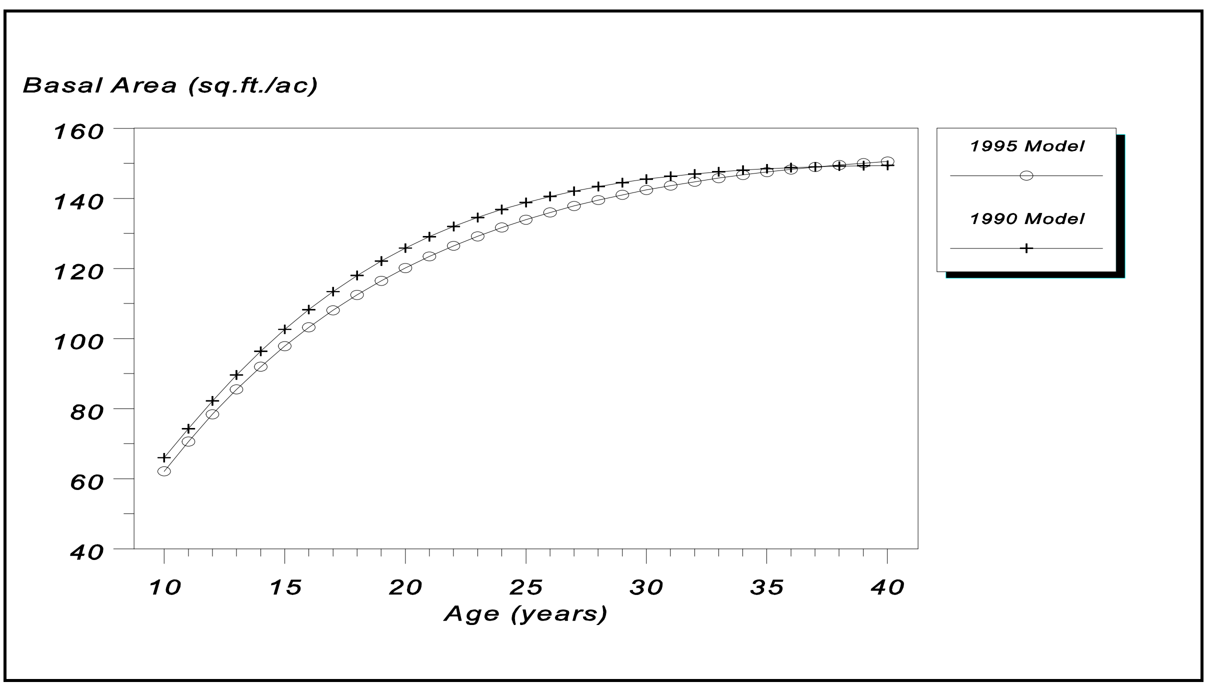

Figure 34. Basal area per acre over age for a height of 30 feet and a density of 500 trees per acre at age 10 in the Lower Coastal Plain region.

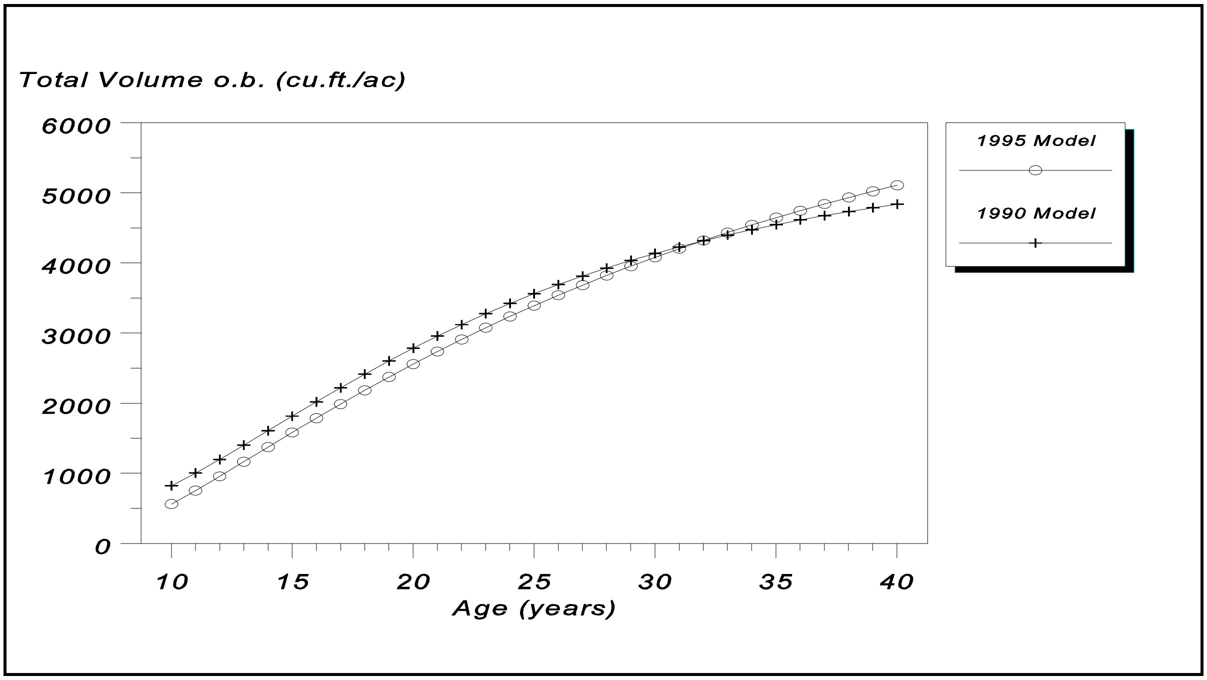

Figure 35. Total volume (o.b.) over age for a height of 30 feet and a density of 500 trees per acre at age 10 in the Lower Coastal Plain region.

8 GROWTH OF THINNED PLANTATIONS

The MS33 thinning study was established to provide data for analysis of the growth of thinned loblolly and slash pine plantations. Fourteen study locations were established in loblolly pine plantations in the Piedmont, Upper Coastal Plain and Lower Coastal Plain regions in South Carolina, Georgia, Florida and Alabama. At each study location, plots were thinned according to three methods: row thinning, selective thinning and a row-select combination. Thinning intensities were set at 33, 40 and 50 percent. The MS33 plots were measured at time of establishment (before thinning) and have been remeasured twice since the thinnings were carried out. The most recent measurement represents between 5 and 7 years post-thinning response. A detailed description and analysis of the MS33 study is given by Brooks (1992).

8.1 Thinned basal area

When thinning intensity is expressed in terms of trees per acre, it is often necessary to obtain an estimate of per-acre basal area removed in the thinning. The following equation can be used to estimate thinned basal area from row, selective or combination thinnings:

$$\frac{BA_t}{BA} = \frac{TPA_r}{TPA} + \left[1 - \left(\frac{TPA_r}{TPA}\right)\right]\left[\left(\frac{TPA_s}{TPA - TPA_r}\right)^{1.2345}\right] \qquad \textbf{(25)}$$

n = 251 R^2 = 0.82 $S_{y.x}$ = 0.035

where:
- BA_t = basal area thinned,
- BA = basal area before thinning,
- TPA = trees per acre before thinning,
- TPA_r = trees per acre removed by row thinning,
- TPA_s = trees per acre removed by selective thinning.

8.2 Thinning growth response

One motivation for thinning pine plantations is the possiblity of increased survival and growth of the residual stand due to decreased inter-tree competition. The MS33 data in combination with the PMRC growth and yield data were used to investigate and model the growth of stands after thinning.

An attempt was made to develop a survival function capable of producing different survival curves for thinned and unthinned plantations. A survival model was successfully fit to the MS33 data only, but it did not predict survival satisfactorily for the PMRC data. An attempt to fit a model to the combined data was not successful. The PMRC survival model (equation **(15)**) fits the MS33 data reasonably well. Table 25 shows average residuals and average absolute residuals in terms of trees per acre for different treatments of the MS33 study.

Table 25: Residual statistics for MS33 survival data predicted with the PMRC survival equation.

Thinning treatment	N	Residual (Obs-Pred)			Absolute Residual		
		Mean	Minimum	Maximum	Mean	Minimum	Maximum
Unthinned	76	-4.2	-85.9	47.1	20.0	1.2	85.9
Row	219	-14.8	-201.5	33.6	18.5	0.1	201.5
Select	211	-3.8	-143.4	37.4	12.9	0.01	143.4
Combination	211	-4.5	-148.3	34.0	11.7	0.06	148.3

It seems logical that if a conscientious job of selective thinning is carried out (e.g. removing trees with obvious defects and removing trees likely to die in the near future) then survival rates should increase. Brooks (1992) discusses this idea and shows support for it from the MS33 data. However, we were not able to detect significant differences in survival by thinning type or intensity. Thus, it is important to keep monitoring the MS33 thinning plots so that we can study this issue further.

When multiple selective thinnings are carried out in a stand, survival rate of the remaining trees should increase. Thus, the survival model presented above is not recommended for use in simulating survival for stands which have received multiple selective thinnings. It may be more reasonable to use a relatively high, constant survival rate after the second and subsequent selective thinnings. This rate should be based on empirical data from stands that have been so thinned.

O'Connor (1935) developed the idea that the basal area of a thinned plantation could be expressed as a proportion of the basal area of an unthinned stand of the same age, dominant height and number of trees per acre (unthinned counterpart). This competition index (CI) expresses the relative degree to which competition affects average tree size in the thinned and unthinned stands:

$$CI = \frac{BA_u - BA_{at}}{BA_u} = 1 - \frac{BA_{at}}{BA_u} \qquad \textbf{(26)}$$

where: BA_{at} = basal area per acre in the thinned stand ,

BA_u = basal area per acre in the unthinned counterpart.

Growth response due to thinning can be expressed in terms of a projected competition index, causing the projected basal area of the thinned stand to approach that of the unthinned counterpart over time. The following equation, proposed by Pienaar (1979), was fit to the MS33 thinning data:

$$CI_2 = CI_1 e^{-\beta(A_2 - A_1)} \qquad \textbf{(27)}$$

Since the MS33 study was not designed to ensure that each thinned plot had an unthinned counterpart with the same number of trees per acre, basal areas for the unthinned counterparts were computed using equation **(16)**. Table 26 shows the parameter estimates and fit statistics for the competition index projection equation.

Table 26: Parameter estimates and fit statistics for the competition index projection equation.

Region	ß estimate	N	R^2	$S_{y.x}$
Piedmont + Upper Coastal Plain	0.076472	227	0.75	0.04
Lower Coastal Plain	0.110521	130	0.41	0.05

Basal area of a thinned stand is projected using the projected competition index (CI_2) as follows:

$$BA_{t_2} = BA_{u_2}(1 - CI_2) \qquad \textbf{(28)}$$

where: BA_{t2} = projected basal area per acre in the thinned stand,

BA_{u2} = projected basal area of the unthinned counterpart.

The use of equations **(25)**-**(28)** is illustrated in Figure 36. The graph first shows an unthinned stand with a site index of 60 feet and 380 trees per acre at age 5. This stand had 350 trees per acre at age 15. Next, a thinned stand of site index

60 feet and 700 trees per acre at age 5 is shown. This stand was thinned selectively from 595 to 350 trees per acre at age 15. Equation **(25)** was used to compute the thinned basal area and basal area per acre after thinning as a function of the number of trees before thinning and the number of trees removed. The after-thinning basal area and the basal area of the unthinned stand, having the same height, age and trees per acre as the thinned stand, were used to compute the competition index with equation **(26)**. This initial competition index was projected in one-year increments up to age 35 using equation **(27)**. The basal area of the thinned stand was then computed with equation **(28)**, using the projected competition index and the projected basal area of the unthinned counterpart. The effect of the reduced competition after thinning is illustrated by the fact that the thinned stand basal area approaches the basal area of the unthinned counterpart over time. The third basal area growth curve shown in Figure 36 is for an unthinned stand which had the same basal area as the thinned stand after thinning. The shaded area represents the growth response due to thinning.

Figures 37 and 38 show the thinned and unthinned basal area growth trends using the Piedmont and Upper Coastal Plain models for different sites. Figures 39 and 40 show the Lower Coastal Plain curves. The trends are similar for the different sites, but on the higher site, the basal area of the thinned stand approaches the unthinned basal area at a faster rate.

When a stand table, either from an inventory or from a diameter distribution model, is available for a plantation before thinning, the thinning can be simulated through the stand table. Examples of stand table thinning algorithms are provided by Grider and Bailey (1984) and by Pienaar et.al. (1996). Once the thinned trees are removed from the before-thinning stand table, the stand table projection method described in section 6.2 can be used for subsequent growth and yield projections.

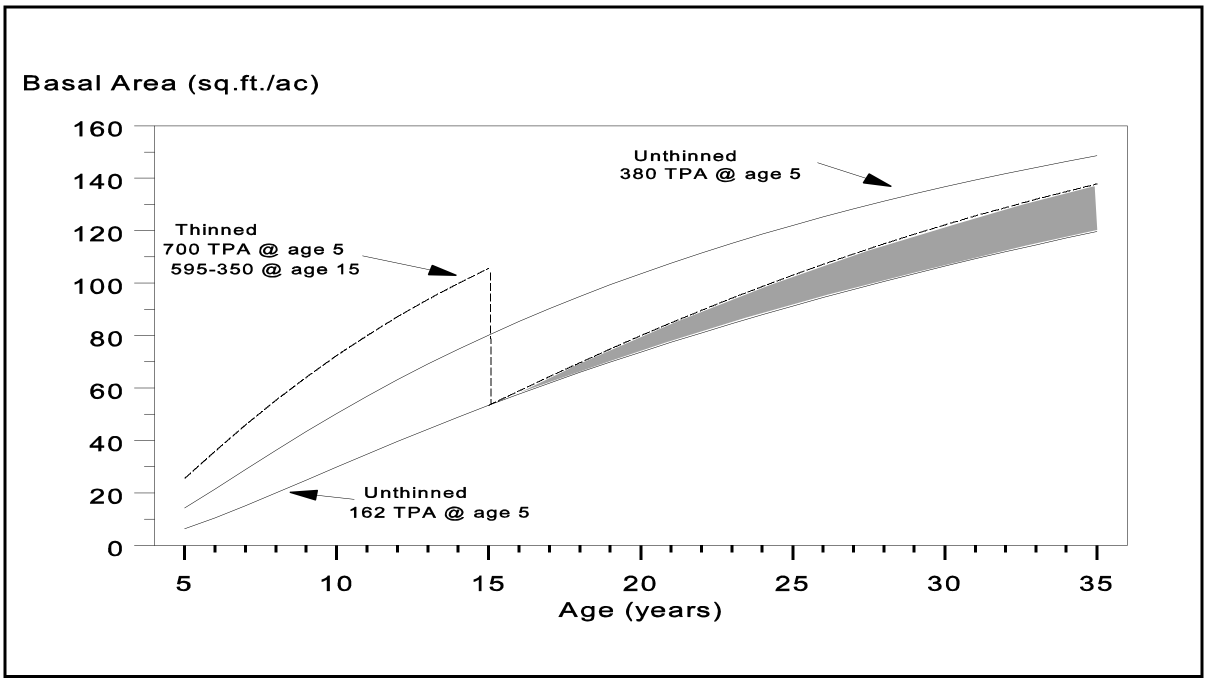

Figure 36. Growth response due to thinning as computed from the competition index.

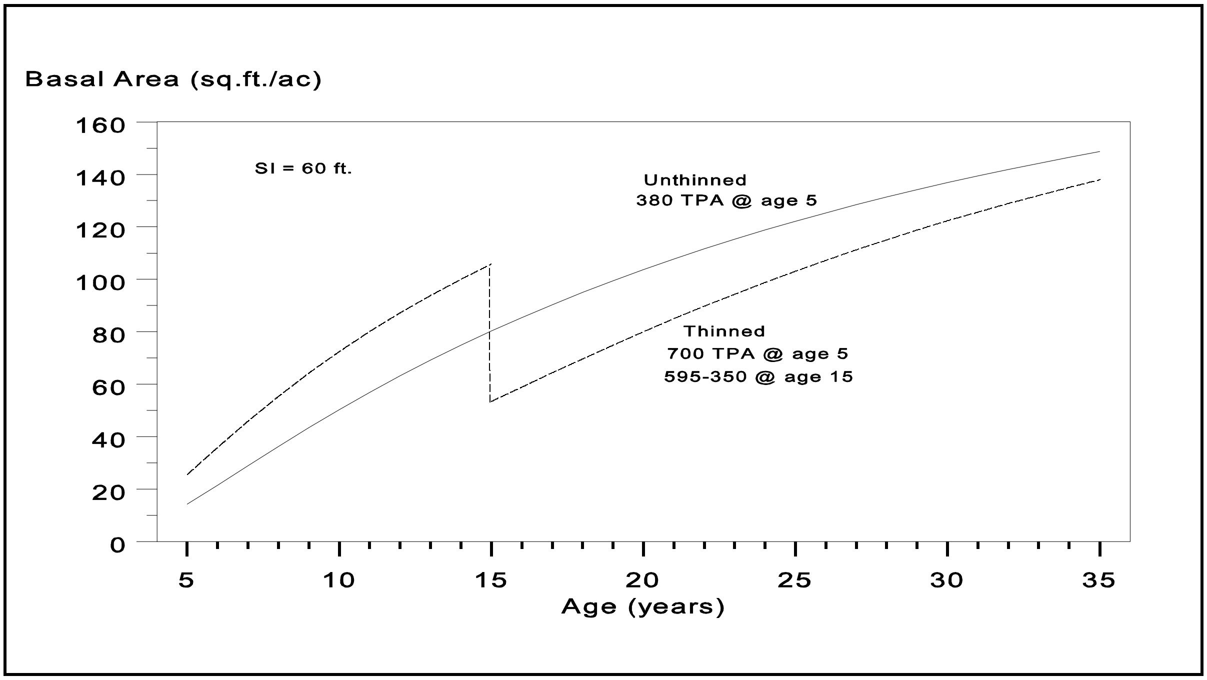

Figure 37. Basal area growth of a thinned stand and an unthinned counterpart with a site index of 60 feet in the Piedmont and Upper Coastal Plain regions.

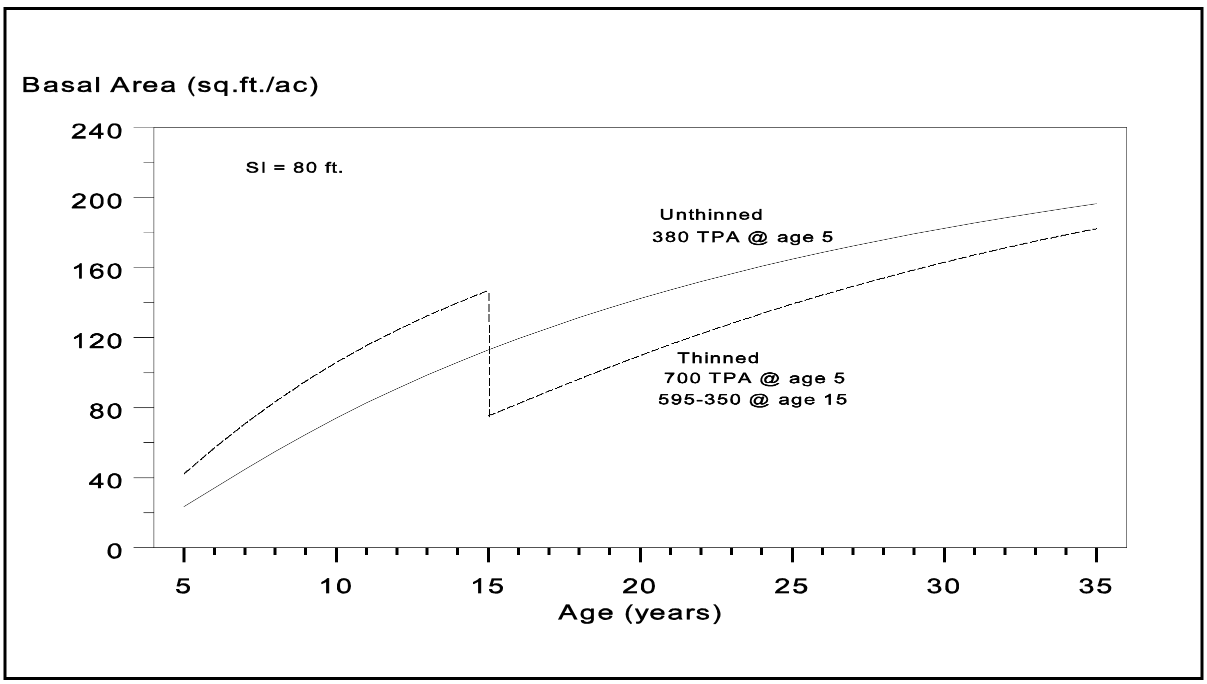

Figure 38. Basal area growth of a thinned stand and an unthinned counterpart with a site index of 80 feet in the Piedmont and Upper Coastal Plain regions..

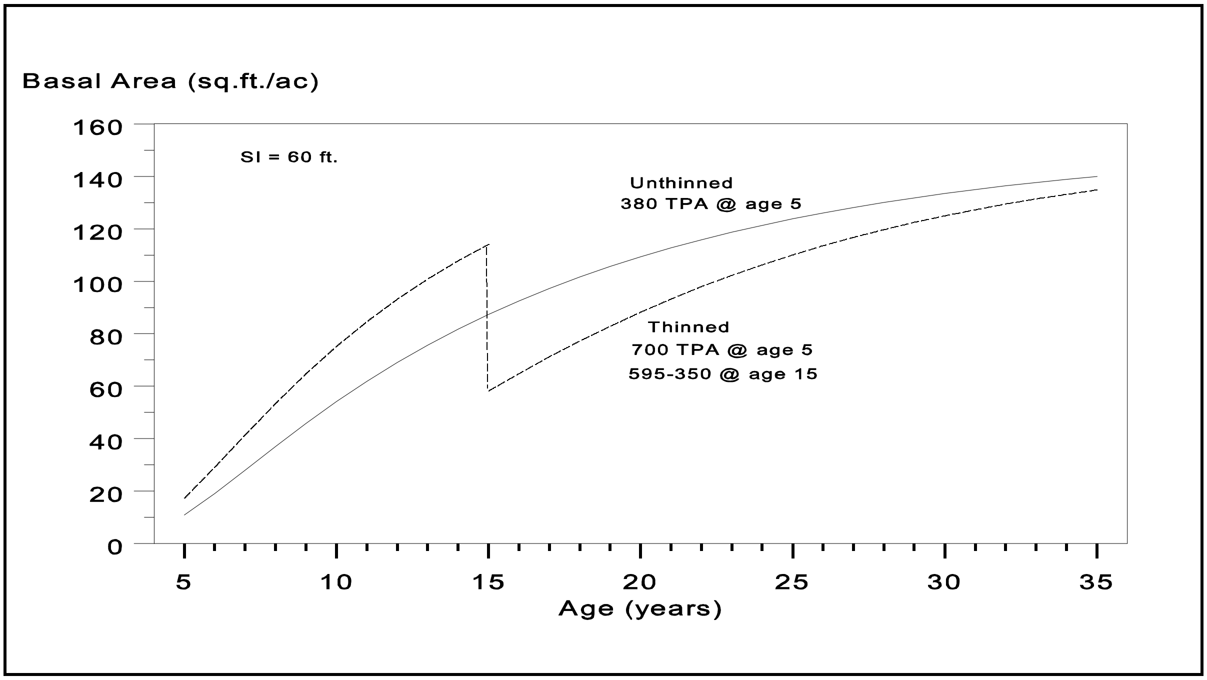

Figure 39. Basal area growth of a thinned stand and an unthinned counterpart with a site index of 60 feet in the Lower Coastal Plain region.

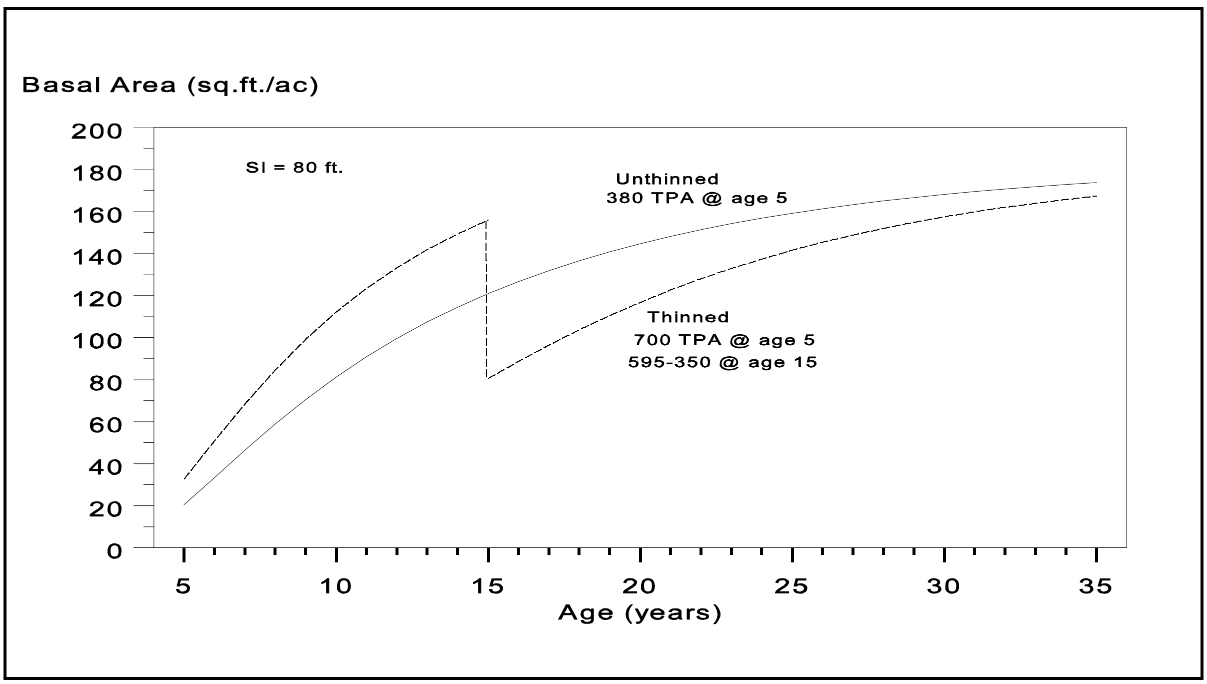

Figure 40. Basal area growth of a thinned stand and an unthinned counterpart with a site index of 80 feet in the Lower Coastal Plain region.

9 GROWTH RESPONSE TO MID-ROTATION FERTILIZATION

Interest in midrotation fertilization with high rates of N and P has increased significantly over the past ten years. In the Southeastern U.S., less than 30,000 acres of loblolly pine stands were fertilized in 1984. This increased to nearly 200,000 acres by 1994. This increase may be, in part, due to results of the NCSFNC Regionwide 13 Study. This study consists of 24 locations, established between 1984 and 1987 in site-prepared loblolly pine plantations across the Southeastern U.S. At each location, two or four replicates of the 12 treatment matrix (0, 100, 200, 300 lbs N/acre in factorial combination with 0, 25, 50 lbs P/acre) were established. Fertilization was carried out at the time of study establishment, resulting in fertilization ages ranging from 10 to 16 years. The most recent remeasurement of the Regionwide 13 study was taken eight years after the fertilization treatment (NCSFNC, 1995). Data from 14 of the locations were used to investigate and model loblolly pine growth response to midrotation N and P fertilization.

9.1 PMRC model validation

The main objective of the fertilization study analysis was to develop additive fertilizer response terms for dominant height and per-acre basal area. In order to accomplish this, it was necessary to validate the PMRC growth and yield models against the unfertilized (control) plots in the Regionwide 13 study. For each control plot, dominant height was projected with equation **(12)**, survival was projected with equation **(15)**, per-acre basal area was projected with equation **(17)** and total per-acre volume and green weight were predicted using equations **(19)** and **(20)**. The average residual, average absolute residual and percent variation explained (PVE) were computed for each model component. The results are shown in Table 27.

Table 27: Residual statistics resulting from use of the PMRC growth and yield models on the Regionwide 13 control plots.

Variable	N	PVE	Residual (Obs - Pred)			Absolute Residual		
			Mean	Minimum	Maximum	Mean	Minimum	Maximum
Dom. Hgt.	360	80.6	0.80	-7.28	8.07	2.15	0.01	8.07
BA / acre	360	90.5	1.80	-20.64	21.25	4.77	0.03	21.25
Trees / acre	360	76.4	31.30	-72.41	147.01	35.54	0.19	147.01
Vol / acre	360	85.4	54.22	-877.60	843.02	226.52	2.60	877.60
Wt / acre	360	85.3	1.48	-24.36	24.08	6.45	0.01	24.36

Examination of the results in Table 27 along with plots of residual versus predicted values for each variable led to the conclusion that the PMRC models were unbiased and sufficiently accurate to proceed with the response modelling exercise. This effort was additionally motivated by a similar residual analysis of the Regionwide 13 fertilized plots.

Residual statistics and graphs for the fertilized plots indicated that the PMRC models were generally biased and tended to underpredict dominant height, per-acre basal area and per-acre yield when no adjustment for fertilization was made.

9.2 Dominant height - response to N and P fertilization

The following adjustment term can be added to the dominant height projection equation to account for midrotation fertilization:

$$R_{HD} = (0.00106\,N + 0.2506\,PZ)\,Y_t\,e^{-0.1096\,Y_t} \qquad \textbf{(29)}$$

N = 4854 $R^2 = 0.92$ $S_{y.x} = 2.27$ ft.

where: R_{HD} = fertilizer response (ft.),

N = lbs of elemental N per acre,

PZ = 1 if fertilized with P,

= 0 otherwise,

Y_t = years since treatment.

No significant difference in fertilizer response by physiographic region was indicated . Figures 41-44 show predicted response and dominant height growth curves for different treatments and treatment ages.

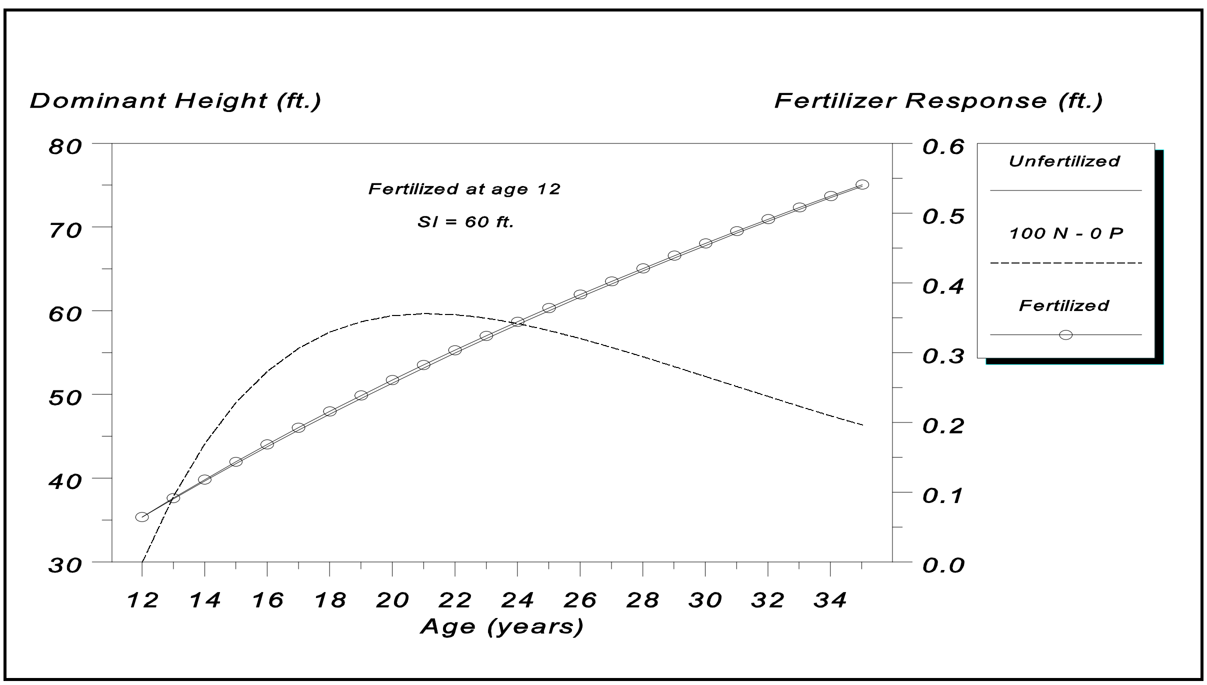

Figure 41. Dominant height growth and fertilizer response for stands with site index of 60 feet, unfertilized and fertilized at age 12 with 100 lbs N and no P.

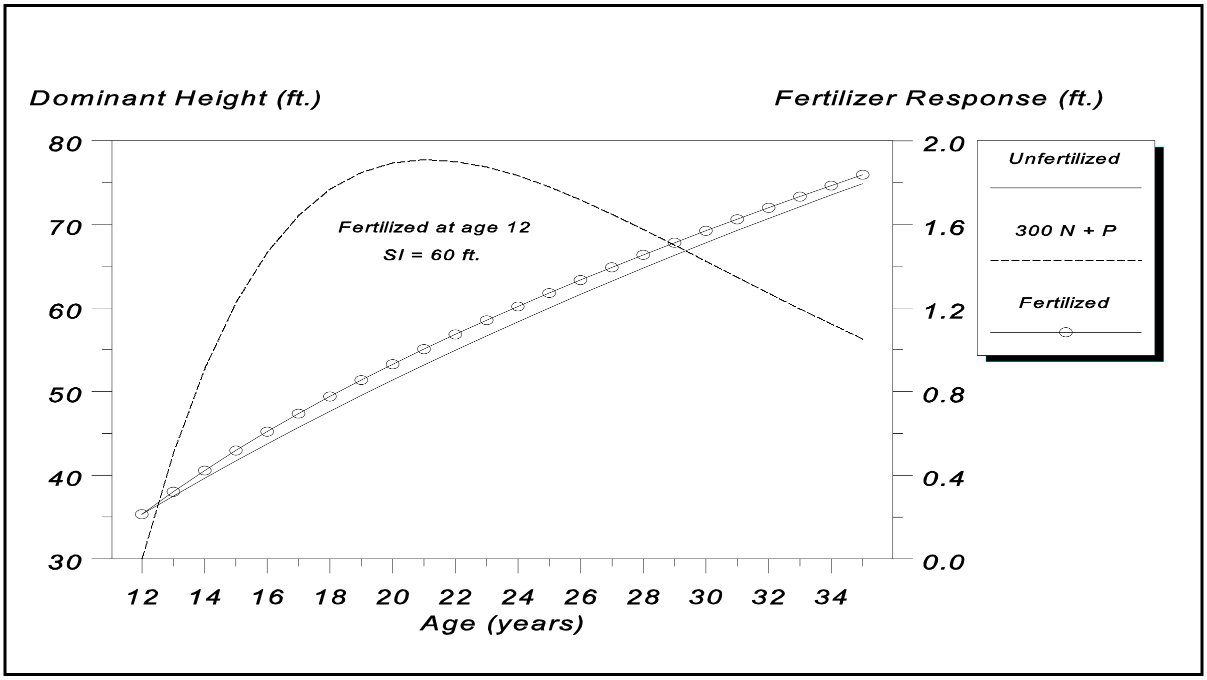

Figure 42. Dominant height growth and fertilizer response for stands with site index of 60 feet, unfertilized and fertilized at age 12 with 300 lbs N with P.

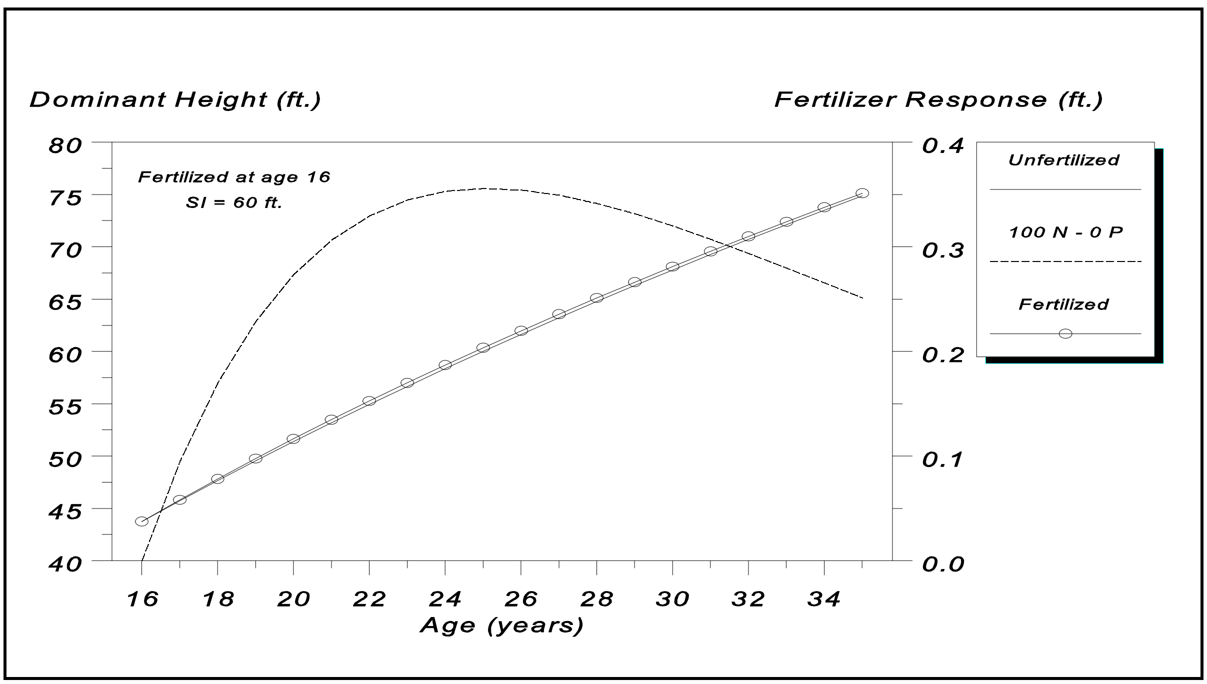

Figure 43. Dominant height growth and fertilizer response for stands with site index of 60 feet, unfertilized and fertilized at age 16 with 100 lbs N and no P.

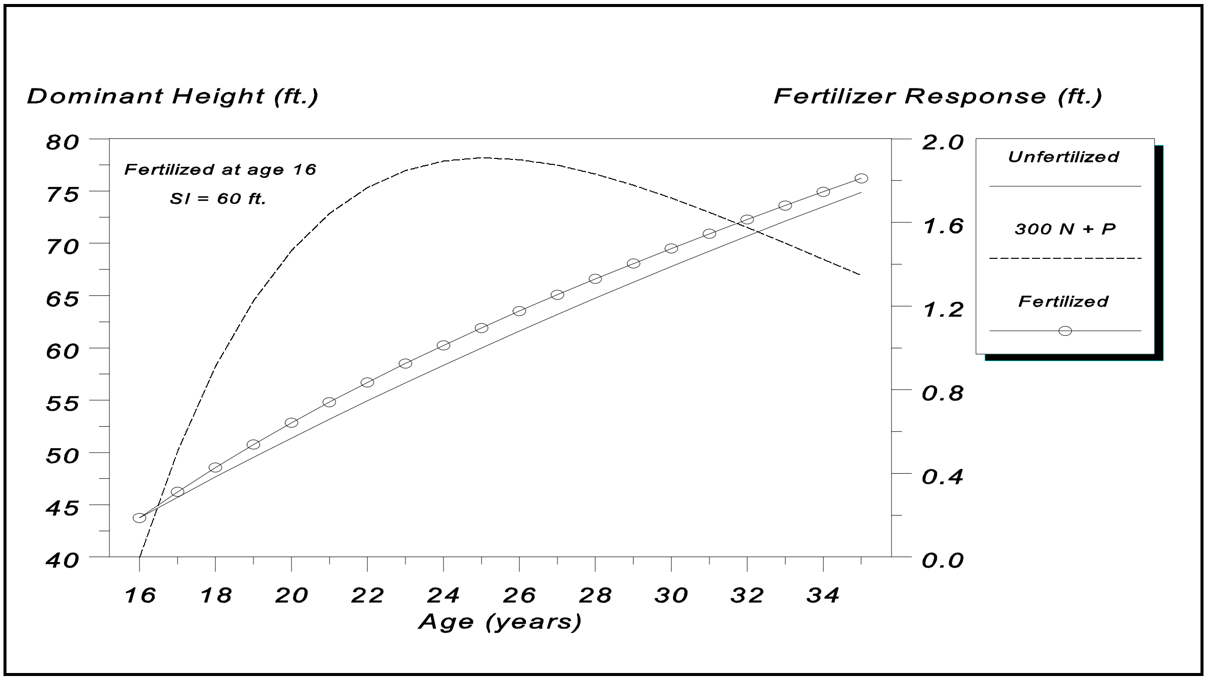

Figure 44. Dominant height growth and fertilizer response for stands with site index of 60 feet, unfertilized and fertilized at age 16 with 300 lbs N with P.

9.3 Per-acre basal area - response to N and P fertilization

An adjustment term of the same form as the height adjustment can be added to the per-acre basal area projection equation to account for midrotation fertilization:

$$R_{BA} = (0.0121N + 1.3639PZ)Y_t e^{-0.2635\,Y_t} \tag{30}$$

N = 3235 $R^2 = 0.86$ $S_{y.x} = 8.72$ ft^2/ac

where:
- R_{BA} = fertilizer response (ft^2/ac),
- N = lbs of elemental N per acre,
- PZ = 1 if fertilized with P,
  = 0 otherwise,
- Y_t = years since treatment.

No significant difference in fertilizer response by physiographic region was indicated . Figures 45-48 show predicted response and basal area growth curves for different treatments and treatment ages.

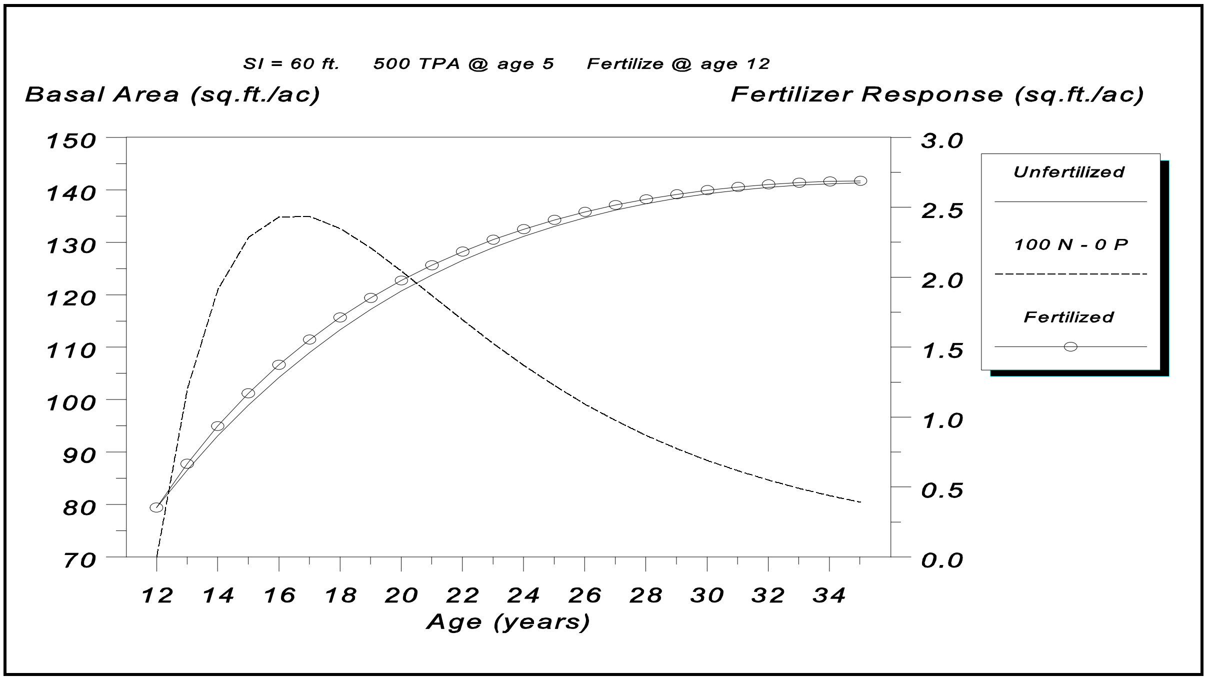

Figure 45. Per-acre basal area growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 12 with 100 lbs N and no P.

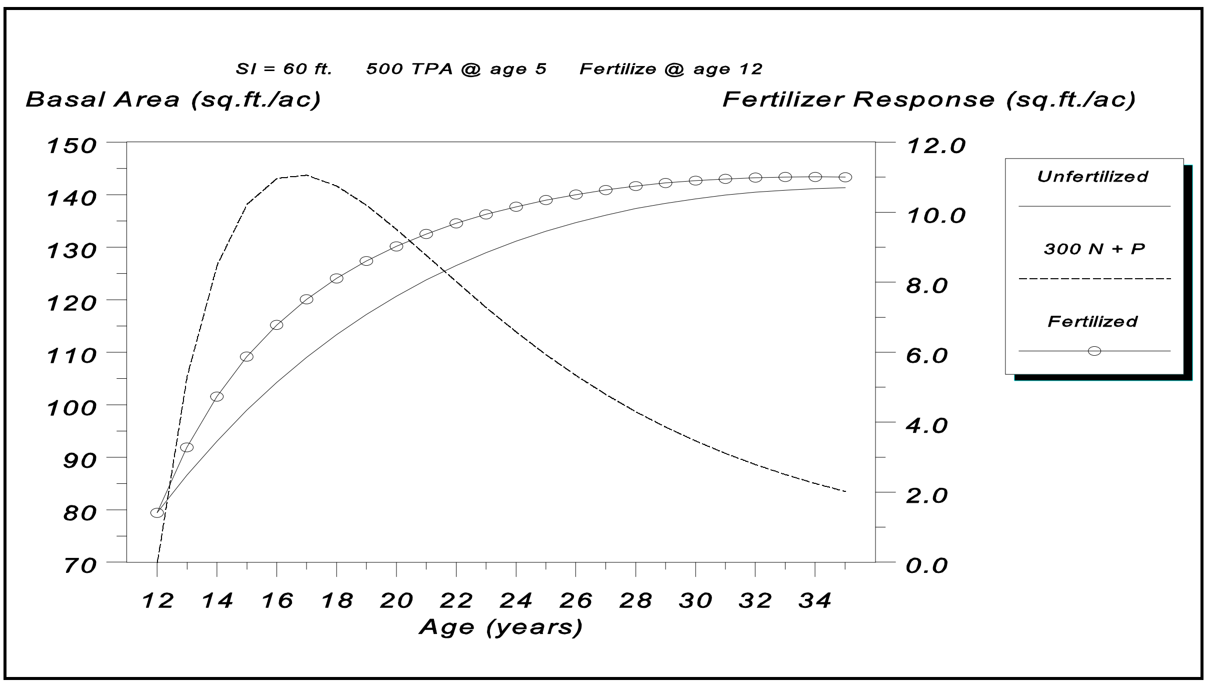

Figure 46. Per-acre basal area growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 12 with 300 lbs N with P.

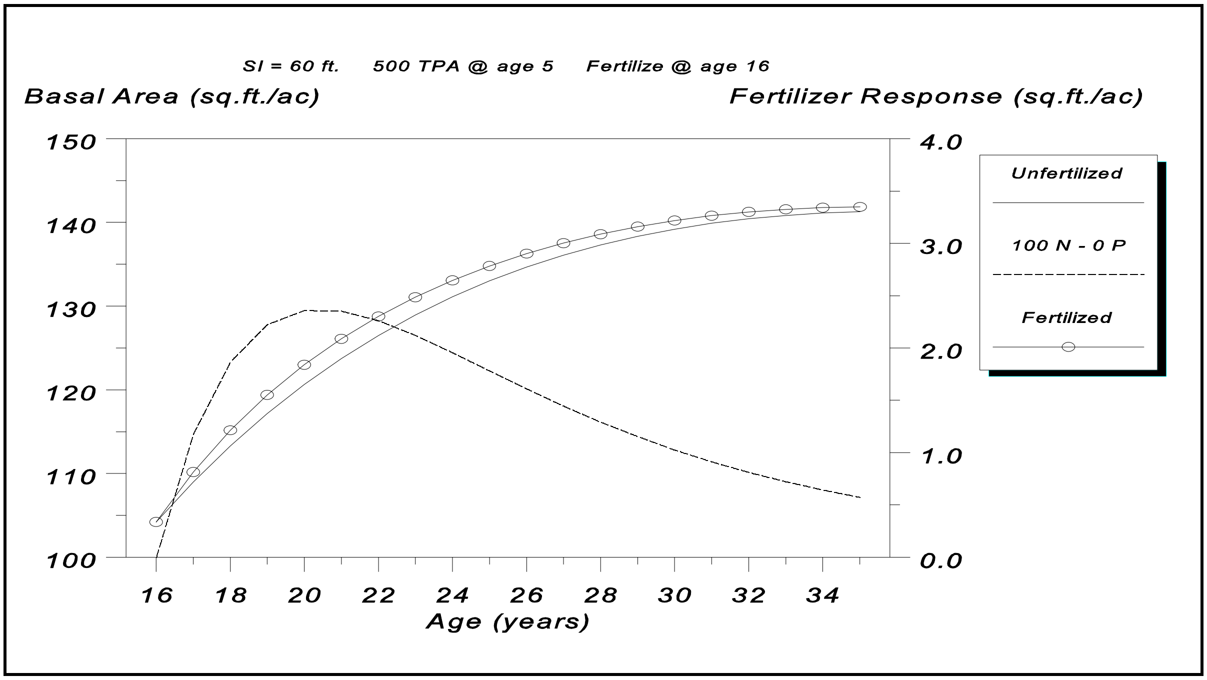

Figure 47. Per-acre basal area growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 16 with 100 lbs N and no P.

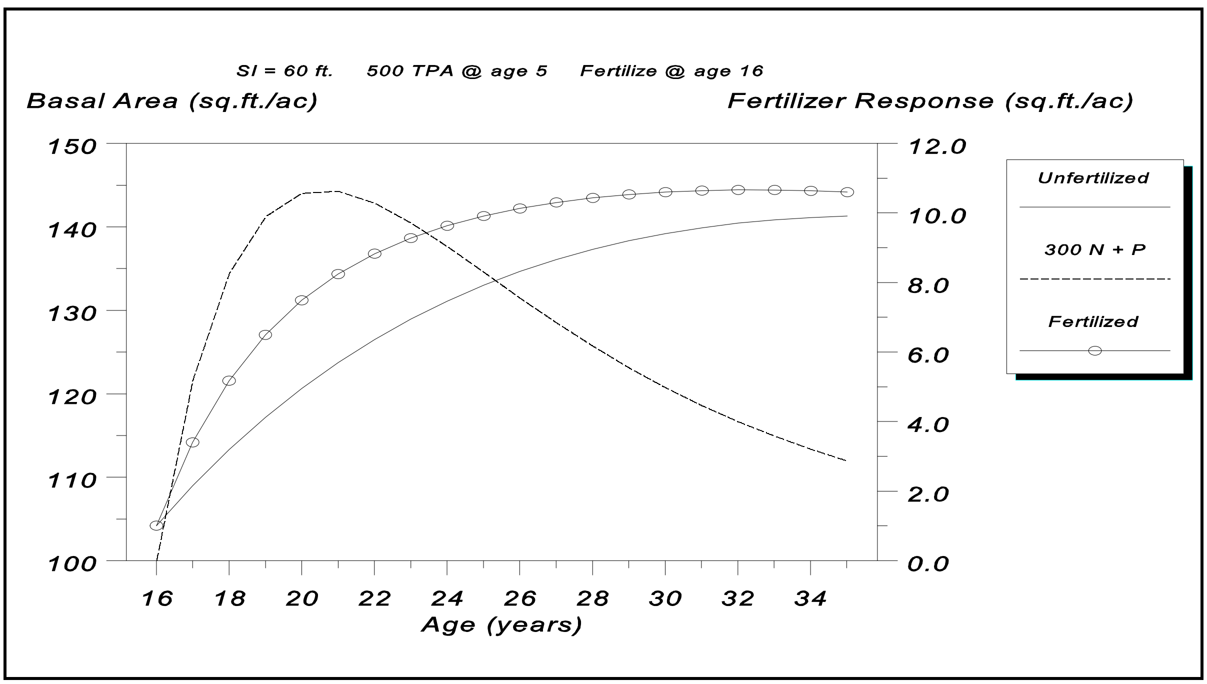

Figure 48. Per-acre basal area growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 16 with 300 lbs N with P.

9.4 Per-acre yield - response to N and P fertilization

Analysis of the Regionwide 13 fertilizer data indicated that an additional adjustment for per-acre yield was not necessary. Since dominant height and per-acre basal area appear in the yield prediction models (equations **(19)** and **(20)**), the effect of fertilization is accounted for in yield prediction by including the adjusted height and basal area. Figures 49-52 show predicted total volume growth curves for various fertilizer treatments and fertilization ages.

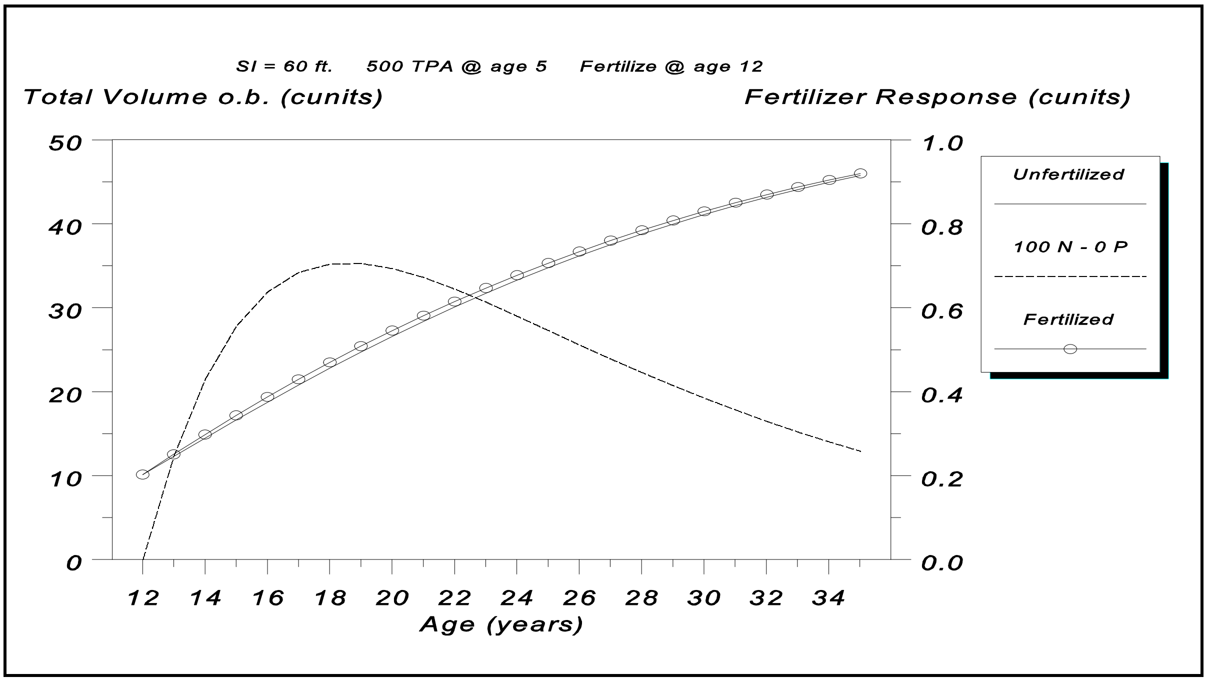

Figure 49. Per-acre total volume growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 12 with 100 lbs N and no P.

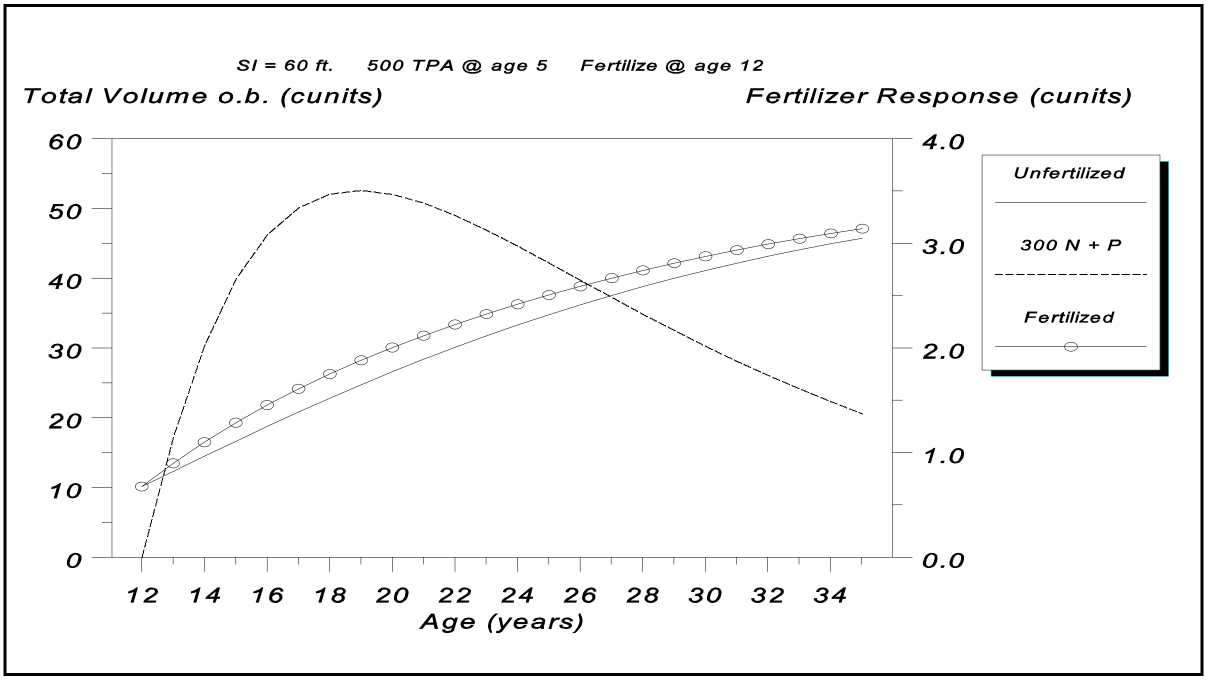

Figure 50. Per-acre total volume growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 12 with 300 lbs N with P.

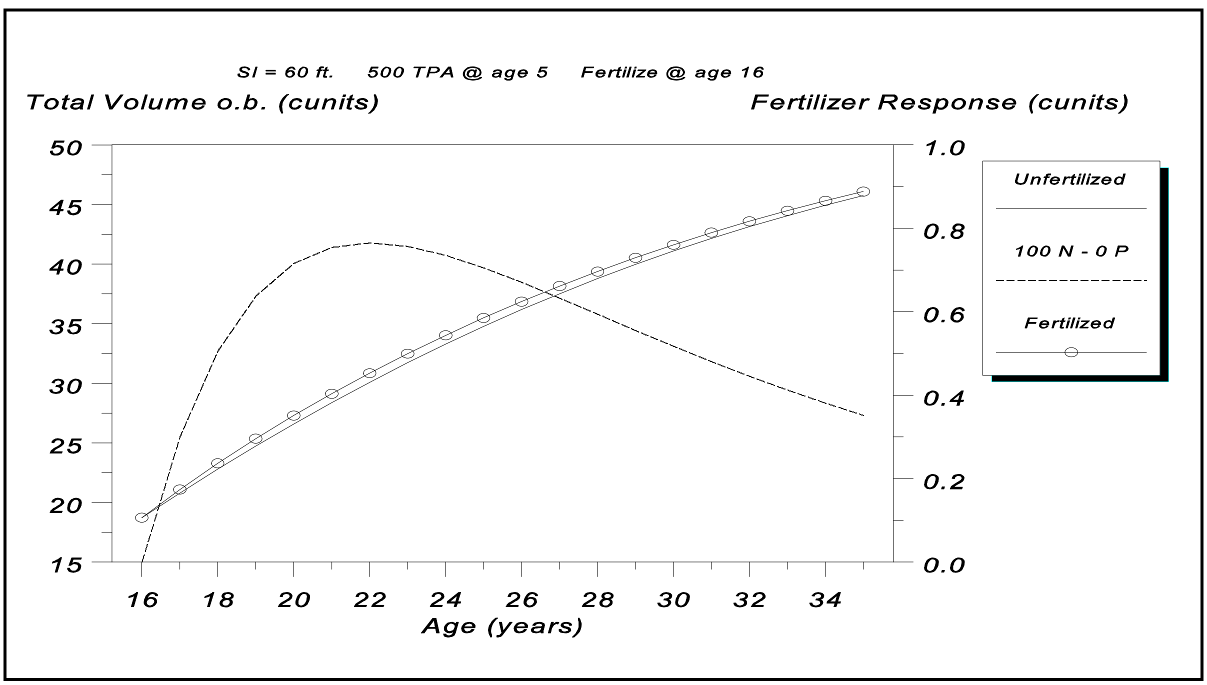

Figure 51. Per-acre total volume growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 16 with 100 lbs N and no P.

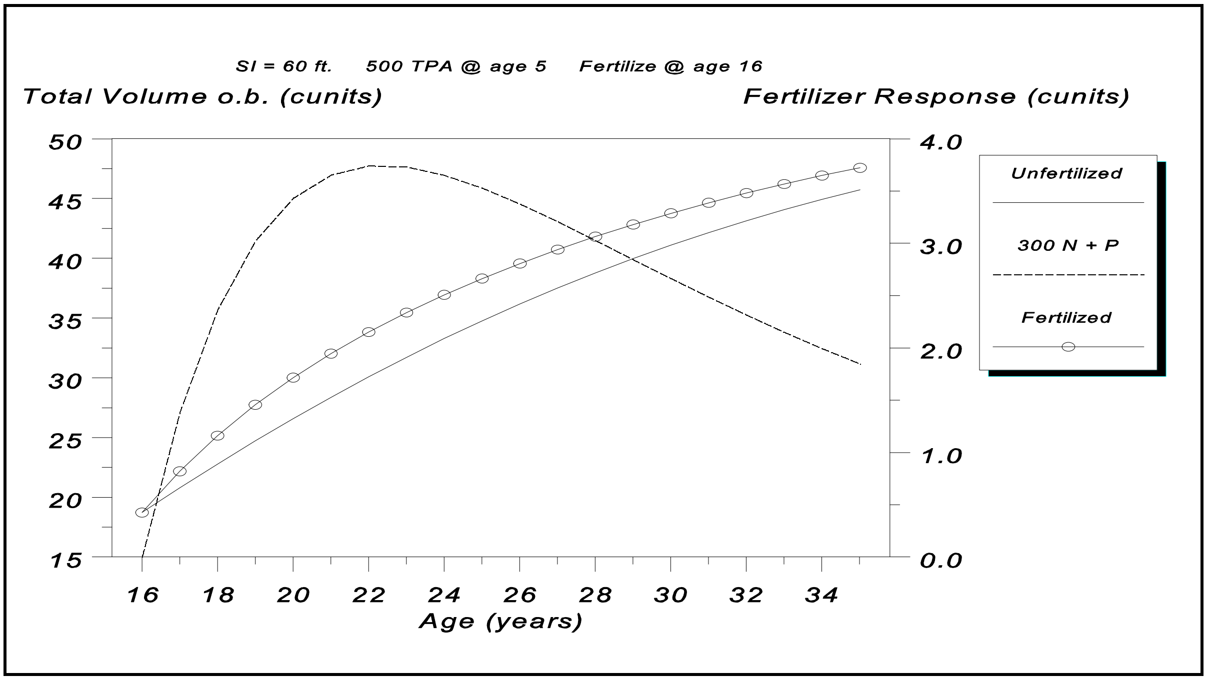

Figure 52. Per-acre total volume growth and fertilizer response for stands with site index of 60 feet, 500 trees per acre at age 5, unfertilized and fertilized at age 16 with 300 lbs N with P.

For those familiar with NC State terminology, the responses presented above can be roughly classified as a type C response for dominant height and a type B response for basal area and per-acre volume.

It should be noted that other models for dominant height and basal area were developed which include age of fertilization as a predictor variable. These models behaved similarly to those presented above, but response to fertilization was greater for stands treated at younger ages. This same results has been found for midrotation fertilized slash pine plantations using the CRIFF database (Bailey et. al., 1996). However, given the small number of locations of the Regionwide 13 study used to develop the fertilizer response models as well as personal communication with the NC State scientists, it was decided that further evidence is needed before including fertilization age in the response models.

Users of the midrotation fertilization response models should realize that these models predict average response to treatment over a very large region. These models, like all empirically parameterized models, are not intended to simulate response in an individual stand. Nor are they designed to dictate or formulate fertilizer prescriptions. It is generally accepted that response to fertilization is highly variable and cannot, as yet, be accurately predicted for a given stand.

9.5 Silvicultural treatment interactions

The models presented above for simulating response to thinning and midrotation fertilization were developed from completely independent databases. It is mathematically possible to use these models to simulate response of thinned stands that have received some type of midrotation gertilization. However, doing so is a beyond the scope of the models and, therefore, users should beware. There is no way of knowing what, if any, interactions apply in thinning-fertilization responses since we have no empirical data with which to evaluate this situation.

10 REFERENCES

Amateis, R.L., Burkhart, H.E. and Burk, T.E., 1986. A ratio approach to predicting merchantable yields of unthinned loblolly pine plantations. For. Sci. 32(2): 287-296.

Bailey, R.L. and Dell, T.R., 1973. Quantifying diameter distributions with the Weibull function. For. Sci. 19: 97-104.

Bailey, R.L., Grider, G.E., Rheney, J.W. and Pienaar, L.V., 1985. Stand structure and yields for site-prepared loblolly pine plantations in the piedmont and upper coastal plain of Alabama, Georgia and South Carolina. Univ. of Ga. Agric. Exp. Stn. Res. Bul. No. 328. Univ. of Ga., Athens, GA. 118 pp.

Bailey, R.L., Martin, S.W. and Jokela, E.J., 1996. Stand-level models for fertilized slah pine plantations. Univ. of Ga., School of Forest Resources PMRC Tech. Rep. 1996-2. Univ. of Ga., Athens, GA. 21 pp.

Borders, B.E., Harrison, W.M., Adams, D.E., Bailey, R.L. and Pienaar, L.V., 1990. Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Florida and Alabama. Univ. of Ga., School of Forest Resources PMRC Res. Pap. 1990-2. Univ. of Ga., Athens, GA. 65 pp.

Borders, B.E., 1994. Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Florida and Alabama: A revised model. Univ. of Ga., School of Forest Resources PMRC Res. Pap. 1994-7. Univ. of Ga., Athens, GA. 23 pp.

Brooks, J.R., 1992. Response to thinning for site-prepared slash and loblolly pine plantations in the Southeast. Univ. of GA PhD Dissertation. Univ. of GA. Athens, GA. 197 pp.

Clutter, J.L. and Allison, B.J., 1974. A growth and yield model for *Pinus radiata* in New Zealand. In: Growth models for tree and stand simulation. R. Coll. For. Res. Note 30. Stockholm, Sweden. p. 136-160.

Clutter J.L. and Jones, E.P., 1980. Prediction of growth after thinning old-field slash pine plantations. USDA For. Serv. Res. Pap. SE-217. 14 pp.

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H. and Bailey, R.L., 1983. *Timber Management: A Quantitative Approach*. John Wiley and Sons, New York. 333 pp.

Grider, G.E. and Bailey, R.L., 1984. A computer simulation model for stand structure, yield, growth, and financial analysis of thinned, site-prepared, slash pine plantations. Univ. of Ga. Coll. of Agric. Exptn. Stn. Res. Bull. 308. Univ. of Ga., Athens, Ga. 90 pp.

Knowe, S.A., 1992. Basal area and diameter distribution models for loblolly pine plantations with hardwood competition in the Piedmont and upper coastal plain. SJAF 16(2): 93-98.

NCSFNC, 1995. Eight-year growth and foliar responses of midrotation loblolly pine to nitrogen and phosphorus fertilization. NCSFNC Rep. No. 33. For. Nutr. Coop., Coll. fo For. Res., North Carolina State Univ., Raliegh, NC. 200 pp.

O'Connor, A.J., 1935. Forest research with special reference to planting distances and thinning. Br. Emp. For. Conf., South Africa. 30 pp.

Pienaar, L.V., 1979. An approximation of basal area growth after thinning based on growth in unthinned plantations. For. Sci. 25(2): 223-232.

Pienaar, L.V., Burgan, T. and Rheney, J.W., 1987. Stem volume, taper and weight equations for site-prepared loblolly pine plantations. Univ. of Ga., School of Forest Resources PMRC Res. Pap. 1987-1. Univ. of Ga., Athens, GA. 11 pp.

Pienaar, L.V. and Harrison, W.M., 1988. A stand table projection approach to yield prediction in unthinned even-aged stands. For. Sci. 34(3): 804-808.

Pienaar, L.V., Harrison, W.M., Burgan T. and Rheney, J.W., 1988. Yield prediction for site-prepared slash pine plantations in the coastal plain. Univ. of Ga., School of Forest Resources PMRC Tech. Rep. 1988-1. Univ. of Ga., Athens, GA. 81 pp.

Pienaar, L.V., Shiver, B.D. and Rheney, J.W., 1996. Yield prediction for mechanically site-prepared slash pine plantations in the southeastern coastal plain. Univ. of Ga., School of Forest Resources PMRC Tech. Rep. 1996-3. Univ. of Ga., Athens, GA. 57 pp.

Reineke, L.H., 1933. Perfecting a stand density index for even-aged forests. Journ. Agric. Res. 46: 627-683.

Sokal, R.R. and Rohlf, F.J., 1981. *Biometry, Second Edition*. W.H. Freeman and Company, San Francisco. 859 pp.